EXPERIMENTAL STUDY OF RESONANT INTERACTIONS OF INSTABILITY WAVES IN AN AIRFOIL BOUNDARY LAYER

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Abstract The present paper is devoted to the detailed experimental study of weakly nonlinear resonant interactions of Tollmien-Schlichting waves in a specially designed 2D non self-similar boundary layer on an airfoil. The influence of the fundamental frequency on the efficiency of the tuned subharmonic resonance is investigated as well as the influence of frequency and spanwise wavenumber detunings. The results are compared with Direct Numerical Simulations based on a vorticity-velocity formulation of the complete Navier-Stokes equations. Good overall agreement is achieved.

1. Introduction

For the design of subsonic, natural laminar flow (NLF) airfoils usually half-empirical transition prediction codes based on linear stability theory (LST) are applied. This approach gives reliable results as long as the streamwise extend of the nonlinear disturbance growth is short in comparison to the linear part. Detailed optimizations of boundary layer parameters with respect to a long laminar run now take advantage of the successive (linear) amplification and damping of Tollmien-Schlichting (TS) waves in a way that the resulting amplitudes just remain below a certain critical threshold. In this case the onset of nonlinear interactions is of significant importance for the extent of the laminar run.

At this stage the disturbances in the boundary layer can still be described as different modes of the frequency-wavenumber spectrum. The basic properties, like amplitude and phase in the y-profiles, remain the same as predicted by linear theory. The interaction of those modes is dominated by so called resonant interactions (see review by Herbert [1] and Kachanov [2]). The classical resonance is the Craik Triad [3], consisting of a 2D fundamental wave with a pair of 3D subharmonic waves with half of the fundamental wave frequency. Subsequent theoretical and experimental studies [4, 5] have shown that this resonance is not only possible for the classical tuned case, where frequency, streamwise and spanwise wavenumber and phase fit. In experiments [5] on a self-similar adverse pressure gradient (APG) flow the highest amplification rates were observed for detuned (mainly with respect to frequency) modes. The double-exponential growth of these modes leads very fast to flow randomization and transition. In previous investigations [6, 7] those systematic studies were extended to the case of a non self-similar boundary layer, which exhibits a rather strong change of the resonance conditions in streamwise direction in comparison to the self-similar one. The present paper is devoted to the comparison of the results obtained in these experiments with DNS.

2. Experimental Setup

The experiments were conducted in the Laminar Wind Tunnel (LWT) of the IAG. The LWT is an open return tunnel with a turbulence level less than $\mathrm{Tu} = 2\cdot 10^{-4}$. The boundary layer measurements were performed on the lower surface of the WW03BL106 airfoil section, which was specially designed to give a constant threshold of a n-factor of $n = \ln(A(f)/A_0(f)) = 6$ (downstream of $s/s_{\max} = 0.3$, $s_{\max} = 604$ mm, arc length measured from the leading edge) at an angle of attack of 2 degree and a Reynolds number of approx. $0.7 \cdot 10^6$. The experiments were carried out at controlled disturbance conditions. The TS-waves were excited by a slit source which was mounted flush to the surface at $s/s_{\max} = 0.13$. The slit is 0.2 mm wide and extends 290 mm in spanwise direction. Below the slit, 116 equally spaced pneumatic tubes are connected to 32 micro loudspeakers which are driven by power amplifiers and a 16 channel signal generator. Independent memory of 4096 points (12 bit resolution) for each channel and external triggering by a quartz based clock enables the generation of disturbances with different frequencies and spanwise wavenumbers. Hot-wire measurements were performed downstream of the slit with a phase locked (with respect to the disturbance generator) data acquisition. Due to the quadratic influence of the velocity on the stability characteristics the velocity ($U_\infty = 18$ m/s) was fixed rather than the Reynolds number (the kinematic viscosity varied between $14.5 \cdot 10^{-6}$ and $16.4 \cdot 10^{-6} \mathrm{m}^2$/s due to temperature effects). The AC-signal of the Dantec 55M10 bridge was band pass filtered and optimal adjusted to the input range of the 12 bit AD-converter by a programmable amplifier. A total of 2^{15} points were sampled at approximately 10 kHz at every measurement station. The time traces are corrected for the influence of the filters by a forward-backward Fourier transform before applying King's

Law to the total signal. The complex TS-wave amplitudes were determined by a final Fourier transform. Spanwise scans and sets of wall normal profiles were performed at several downstream positions. All necessary base flow parameters were obtained, including stability characteristics for 2D and 3D TS-waves in a range of frequencies from 255 to 610 Hz (see figures 1 and 2).

3. Experimental Results

In a first step resonant triplets were excited consisting of a 2D fundamental wave with an amplitude of approximately 0.06% of u_e at 20 mm downstream of the disturbance source and a pair of 3D subharmonic waves with an amplitude of about one order below the fundamental one [6]. The resonance condition was always satisfied at the position of the disturbance source. Despite the rather strong downstream variation of the resonant subharmonic spanwise wavenumber, resonant growth is present in a wide range of tuned and detuned frequencies and spanwise wavenumbers in general consistence with experiments [5, 8] in a self-similar APG boundary layer.

It was found, as shown in figure 3, that the resonant amplification is the strongest at high fundamental wave frequencies (which are most unstable at the initial section of the airfoil) and subsides quickly with decreasing frequency. This phenomenon seems to be explained by the frequency variation of the dispersion characteristics of the base flow due to its essential non self-similarity. The dependence of the resonant amplification on the initial phase shift led to an anti-resonance regime where suppression of the subharmonic mode is observed as shown in figure 4.

Measurements with frequency detuning [7] showed that for high fundamental frequencies the strongest amplification occurs at small positive frequency detunings, while at low frequencies at very large detunings (about 80% of the subharmonic frequency). For fundamental frequencies $f_1 \geq 400$ Hz the maximum amplification of the subharmonic mode occurred in a rather narrow frequency range close to 300 Hz for the excited subharmonic mode, shown in figure 5. This finding let to a joint interaction regime of a quasi-subharmonic 3D wave pair with two 2D fundamental waves. It was shown that a superposition principle is satisfied which is leading to a very rapid growth of quasi-subharmonic waves with increments, which are as large as (or even somewhat greater than) those observed in the same two resonances when they occur separately.

The subharmonic spanwise wavenumber detuning was shown to influence significantly the resonant amplification of subharmonics, while the tuned regimes still dominate over the corresponding detuned ones (figure 6). The character of this influence correlates with the streamwise variation of the base flow properties.

4. Comparison with Direct Numerical Simulations

The DNS are based on the vorticity-velocity formulation of the complete Navier-Stokes equations for incompressible flat-plate flow with streamwise pressure gradient. 6^{th}-order compact FDs are employed for the streamwise and wall-normal direction, and a Fourier spectral representation for the spanwise direction, with RK4-O4 time stepping. The disturbances are introduced at the same position as in the experiment at $\Delta s = 0$ mm by time wise periodic blowing and suction within a spanwise disturbance strip at the wall. For details of the method see [9, 10]. Simulations were made with the average Reynolds number and kinematic viscosity obtained from all measurements. The DNS disturbances are determined by matching the amplitudes at $\Delta s = 20$ mm to the ones measured at this position.

Experimental and DNS amplification curves for tuned resonance regimes are shown in figure 3. The comparison shows good quantitative agreement up to the end of the parametric stage at $\Delta s \approx 130$ mm for a wide rage of fundamental frequencies.

Figure 4 shows the subharmonic growth rates for different experimental and DNS regimes with the same fundamental frequency $f_1 = 510$ Hz (sub only: no excitation of fundamental wave). Except for the anti-resonance regime all other subharmonic amplitudes coincide within each regime. The difference in the anti-resonance regime is attributed to the inaccuracy of the experimental determination of the exact phase shift within a adequate time frame. The experimentally found phase shifts varied from 146° for $f_1 = 610$ Hz to 153° for $f_1 = 409$ Hz whereas the DNS phase shifts were found to be approx. 4° higher. Therefor the streamwise length of suppression of the measured subharmonic is shorter than the one found in DNS. Upstream of the suppression both growth rates are again identical.

The dependence of the effective subharmonic mode amplification on frequency detuning is shown in figure 5. The amplitudes were measured close to the disturbance source $\Delta s_1 = 40$ mm and farther downstream at $\Delta s_2 = 120$ mm (160 mm for $f_1 = 255$ Hz). The same positions were used for the determination of the amplification found in DNS. For the highest fundamental frequencies ($f_1 = 510$ and 610 Hz) very good quantitative agreement was found over the entire range of investigated detunings ($\pm 80\%$). The different amplifications for lower fundamental frequencies are showing good qualitative agreement by matching the excited subharmonic frequency where the maximum resonant interaction is observed in the experiment. The quantitative deviation can be explained by a slight difference in the growth rate of the subharmonic modes leading to lower intergral amplitudes in DNS.

The influence of spanwise wavenumber detunings on the resonant amplification of subharmonic waves was investigated in detail for the strongest resonant interaction ($f_1 = 610$ Hz) in a range of $\pm 100\%$ of the resonant spanwise wavenumber (-100% is equivalent to 2D excitation). The estimated values of the subharmonic growth rates are presented in figure 6 versus the spanwise wavenumber for three streamwise positions. In general the behavior is similar with good agreement for positions closer to the disturbance source and for the resonant or higher spanwise wavenumbers. The cause for the differences and especially for $\beta = 0.2$ rad/mm have yet to be determined.

5. Conclusion

Weakly nonlinear interactions of TS-waves have been studied under controlled disturbance conditions in an essentially non self-similar APG boundary layer on an airfoil. Despite rather strong downstream variation of the resonant subharmonic spanwise wavenumber the resonant growth is found in a wide range of tuned and detuned frequencies and spanwise wavenumbers in general consistence with previous experiments [5, 8]. The obtained DNS results showed in a wide range of experimental regimes an excellent agreement. Minor differences were found only in the anti-resonance regimes cause by uncertainties in the determination of the experimental phase shift for maximum suppression of the subharmonic mode and in the comparison of the estimated values of the subharmonic growth rates for different spanwise wavenumbers.

Acknowledgments

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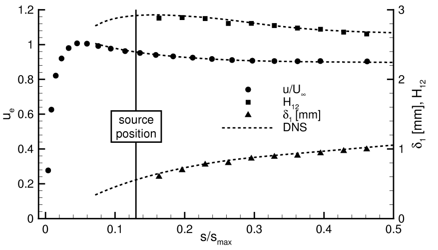

Figure 1. Distribution of velocity, shape factor and displacement thickness .

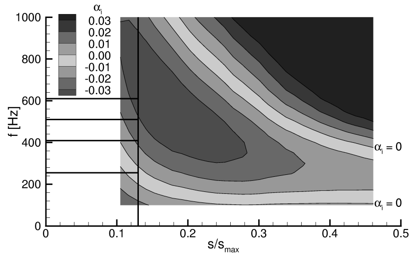

Figure 2. 2D stability diagram with position of source and investigated fundamental frequencies.

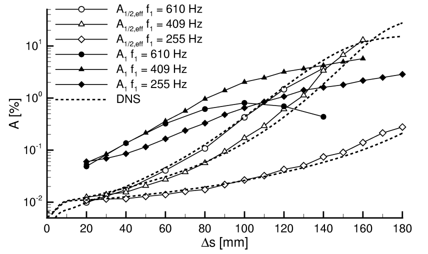

Figure 3. Amplification curves for investigated fundamental frequencies in tuned regimes.

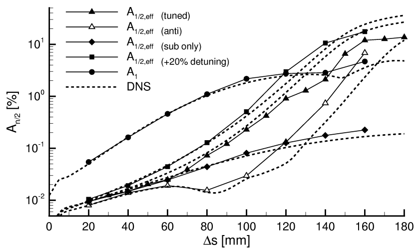

Figure 4. Amplification curves for tuned, anti-resonance, subharmonic only and maximum resonance detuned regimes.

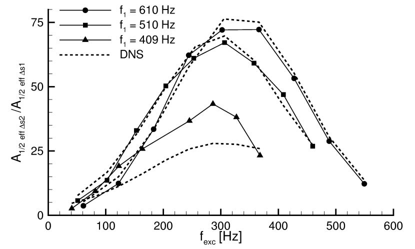

Figure 5. Effective mode amplification factors vs. frequency of excitation.

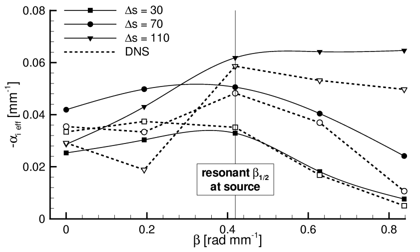

Figure 6. Influence of wavenumber detuning on subharmonic growth rates.