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DNS OF TRANSITION MECHANISMS AT MACH 6.8 – FLAT PLATE VS. SHARP CONE

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Abstract. Spatial Direct Numerical Simulations are carried out to study laminar-to-turbulent transition mechanisms in hypersonic boundary layers at flight conditions (hot flow). The goal is to find possible differences between the flat-plate and sharp-cone boundary layer. Linear Stability Theory shows that 2-d 2nd-mode disturbances reach relevant N-factors much earlier on the cone. This does not hold for 3-d disturbances, so fundamental breakdown is favoured on the cone, whereas at the flat plate oblique breakdown is the dominant mechanism. Direct Numerical Simulations show that the heating and the shear stresses at the wall are much stronger in case of fundamental breakdown and therefore on the cone. In principle the respective mechanisms work in the same manner on flat plate and cone. However, fundamental breakdown is accelerated by the decreasing propagation angle of the secondary 3-d wave on the cone, whereas oblique breakdown is delayed by the weaker growth of the steady longitudinal vortex mode.

Key words: boundary layer, hypersonic flow, laminar-turbulent transition, Direct Numerical Simulation, Linear Stability Theory

1 INTRODUCTION

Laminar-turbulent transition in boundary layers of bodies in supersonic flow does not only have strong influence on shear stresses and heat flux, but also on other flow phenomena like shock/boundary-layer interaction and flow separation, and can therefore influence the global flow field and the aerodynamic drag substantially. Hence a basic understanding of transition in high-speed boundary layers is important. Because of the complexity of the underlying physical mechanisms examinations of geometrically simple but generic bodies like flat plates or sharp cones are necessary at first.

Investigations using Linear Stability Theory (LST) deliver an overview of the amplification of small-amplitude disturbance waves. Mack[1] and Malik & Spall[2] performed extensive stability investigations for the sharp cone and illustrated the differences to the flat plate. The most important results are presented in section 3. Parabolized Stability Equations (PSE) allow for non-parallel effects and weakly nonlinear stages; a sensitivity of the results with regard to the accuracy of the base-flow solution has been found (e.g.[3]). In the rare wind-tunnel experiments transition is usually caused by uncontrolled background disturbances in the oncoming flow ([4,5]). For an in-depth interpretation and careful comparisons with stability theories and DNS, experiments in quiet tunnels with controlled disturbance input are essential. A first controlled experiment on a cone in hypersonic flow has been performed by Maslov et al.[6]

All present investigations of other research groups on nonlinear breakdown phenomena in super- and hypersonic flow are concerned with cold boundary layers at wind-tunnel conditions. In this work transition is examined at flight conditions in the atmosphere.

2 NUMERICAL METHOD

The flow is considered in an integration domain on the flat plate and the sharp cone (7° semivertex angle), respectively, not containing any shock wave induced by the leading edge. Fig. 1 shows a sketch of the integration domain as used on the cone.

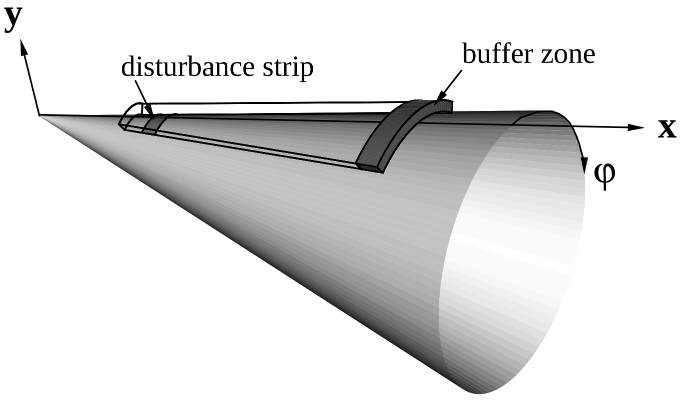


Figure 1: Integration domain

In a disturbance strip at the wall controlled disturbances are excited by timewise periodic and simultaneous blowing and suction. At the end of the integration domain a buffer domain is appended, in which all disturbances of the flow are smoothly damped to zero in order to avoid reflections from the outflow boundary. The development of the disturbance waves is computed by solving the complete Navier-Stokes equations in conservative form, solved in a disturbance formulation. The base flow is also a Navier-Stokes solution with an upper boundary condition based on spatial conical characteristic conditions. The fluid is considered as nonreacting, perfect gas, for which the thermodynamic equation of state is valid. All variables are nondimensionalized with respect to free-stream values. For the wall temperature we use $\vartheta \cdot \partial T/\partial y = \epsilon \sigma T^4$ with the Stefan Bolzmann constant σ and the emissivity ϵ.

The discretization in streamwise (x-) and wall-normal (y-) direction is done by 6th-order compact finite differences with high modal resolution, which are used in a splitted form.[7] In spanwise/azimuthal (z/φ-) direction a Fourier spectral approach

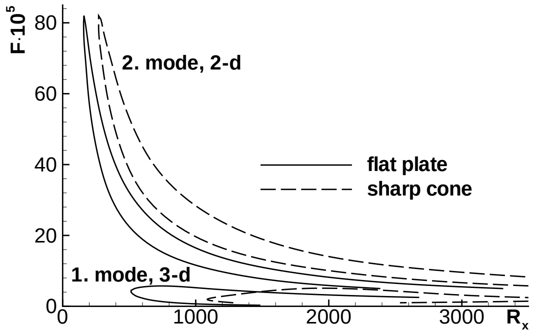


Figure 2: Neutral stability curves; flight conditions, cooled wall ($T_w = 975$ K); for 3-d modes $\beta = 2.165 \cdot F \cdot Re$ (plate) and $n = 9 \cdot F \cdot Re$ (cone), respectively

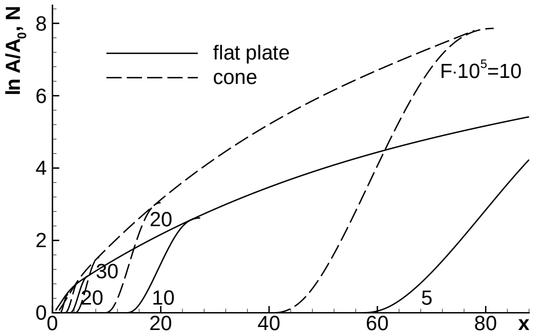


Figure 3: Individual-disturbance amplification curves and envelope curves; 2-d disturbances, various frequency parameters F; $N = \ln A/A_0$

is used. The time integration is performed by a 4-step Runge-Kutta scheme of 4th-order accuracy.

3 STABILITY PROPERTIES

The transition in hypersonic boundary layers is not independent of temperature effects. For our simulations we choose flight conditions, i.e. “hot” flow ($T_\infty = 220$ K) and radiation-adiabatic cooled wall ($T_W \approx 0.5 \cdot T_{rec}$). Under these conditions acoustic (2nd-mode) disturbances are destabilized; the most unstable of these modes are two-dimensional.

Using the Mangler-Levy-Lees transformation and neglecting body curvature the compressible boundary layer on the sharp cone at zero angle of attack can be put down to the self-similar boundary layer on the flat plate (with $x_{cone} = 3 \cdot x_{plate}$). Under these conditions known Linear-Stability-Theory results show that the unstable region for 2-d disturbances starts three times further downstream on the cone (see Fig. 2; note the scaling $R_x = \sqrt{x \cdot Re}$). However, the unstable region for each frequency is three times longer and therefore the integral amplification rates shown in Fig. 3 are significantly higher. It can also be seen that the frequencies relevant for transition are higher for the cone.

It is not possible to perform an equally simple comparison for 3-d waves, because due to the body divergence of the cone the propagation angle of 3-d waves gradually reduces with downstream distance. This leads to the important fact that 3-d waves tend to become more 2-d.

4 RESULTS OF DNS

Table 1 shows the parameters of the simulation cases discussed here (for other basic DNS investigations on resonance mechanisms see[8]). The Mach number at the boundary-layer edge is $Ma_e = 6.8$ in all cases. $A_{(h,k)}$ is the disturbance amplitude for mode (h, k); thereby (h, k) denotes a wave with frequency $h \cdot F$ and the spanwise/azimuthal wavenumber $\pm k \cdot \beta$; $F = 2\pi f^* L/(u^*_\infty Re_L)$, $\beta = \beta^* L$, $L = 17.49$ mm, $Re_L = 10^5$. For the cone the local azimuthal wavenumber is defined by $\beta = n/r$ with $\beta_0 = n/R_0$ and an integer $n = \beta r =$const., where r denotes the radial distance of a point from the cone axis; thus $\beta = \beta_0 R_0/r$. Going downstream, the azimuthal wave number and the propagation angle reduce with the increasing radius.

Figure 4 shows the development of the maximum disturbance amplitudes (over

	oblique-type		fundamental-type	
body	plate	cone	plate	cone
$F \cdot 10^5$	2.0	2.0	10.0	5.0
β	4.33	–	11.0	–
n	–	18	–	54
$A_{(1,0)}$	–	–	0.01	0.01
$A_{(1,1)}$	0.005	0.005	0.0035	0.0035
ϵ	0.85	0.88	0.21	0.44

Table 1: Simulation parameters

y) of u' for the oblique-type transition scenario. The disturbed symmetrical pair of waves (1,±1) has vorticity-mode type (first mode) and the same frequency parameter $F = 2 \cdot 10^{-5}$ and obliqueness angle 60° (referred to the location of the disturbance strip) in both cases. The characteristic nonlinearly generated steady longitudinal vortex mode (0,2) has smaller amplification rates in the cone case, although the growth of the generating mode is nearly the same. This is not caused by curvature or body divergence effects since simulations with different cone angles showed nearly the same result. It may be the effect of the slower-growing boundary layer which possibly hinders the growth of vortices. Besides that, the mode development and the physical structures are quite similar.

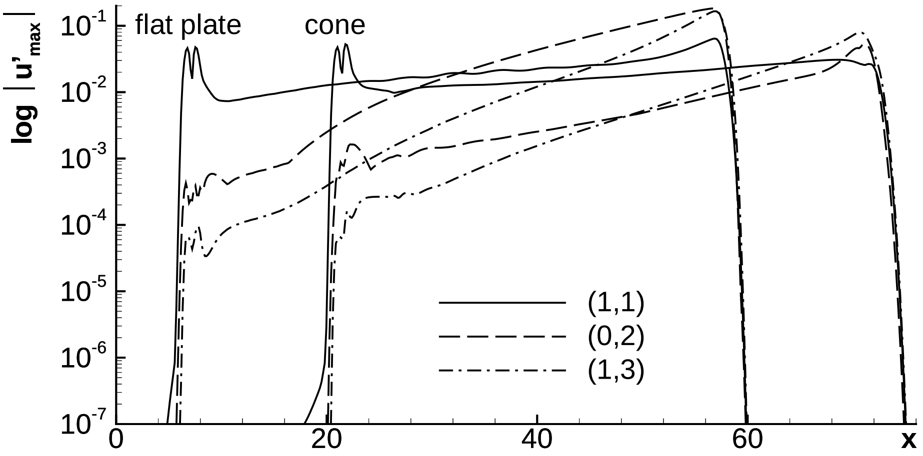


Figure 4: Downstream development of the $|u'|$-disturbance amplitudes; oblique-type transition

In the fundamental-type transition scenario an acoustic 2-d primary disturbance (1,0) and a secondary 3-d mode (1,±1) with the same frequency, but small amplitude, are excited. The frequency parameters are $F = 5 \cdot 10^{-5}$ for the flat plate and $F = 10 \cdot 10^{-5}$ for the cone, respectively, so that the resulting N-factors are roughly comparable (cf. Fig. 3). Again, the maximum amplitudes are displayed in Fig. 5. The 2-d primary wave (1,0) grows due to the primary instability. At a threshold amplitude, the phase speed of the secondary 3-d wave (1,±1) starts to synchronize with the primary wave; after adaptation resonant growth of (1,±1) and (0,1) takes place. Due to the stability properties of the 2-d 2nd-mode disturbance the threshold amplitude is reached much earlier at the cone, as can be seen in Fig. 5 (the disturbance strip is in both cases located at the beginning of the unstable region

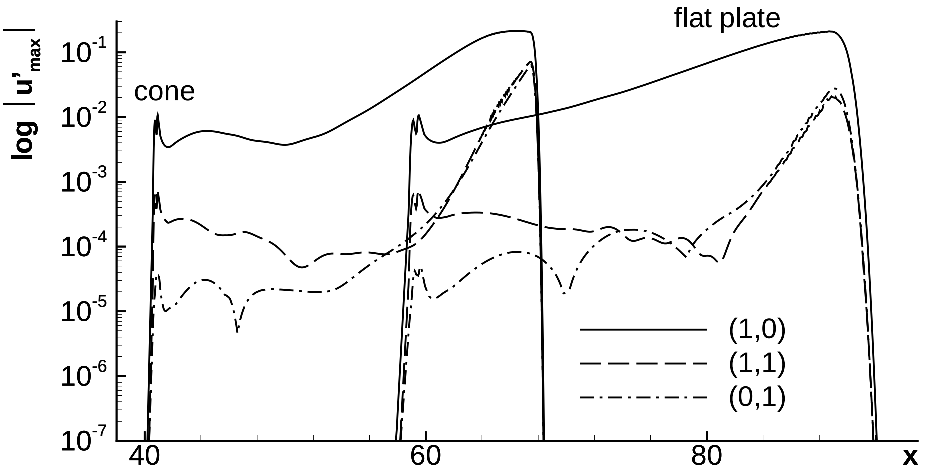
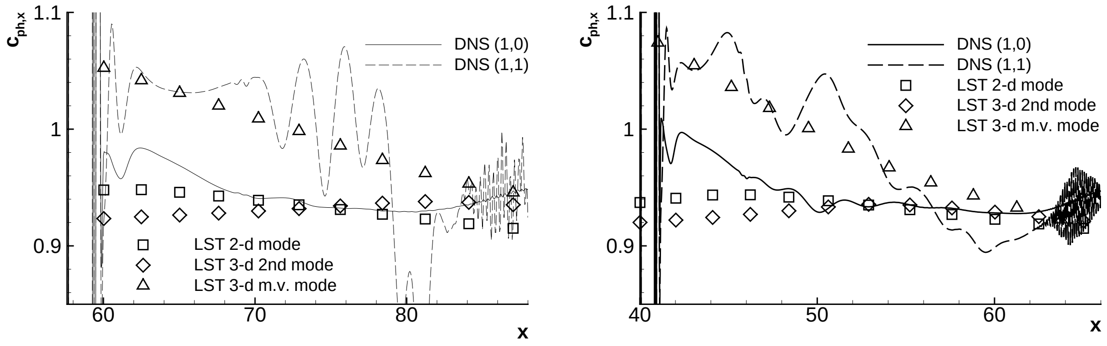


Figure 5: Downstream development of the $|u'|$-disturbance amplitudes; fundamental-type transition

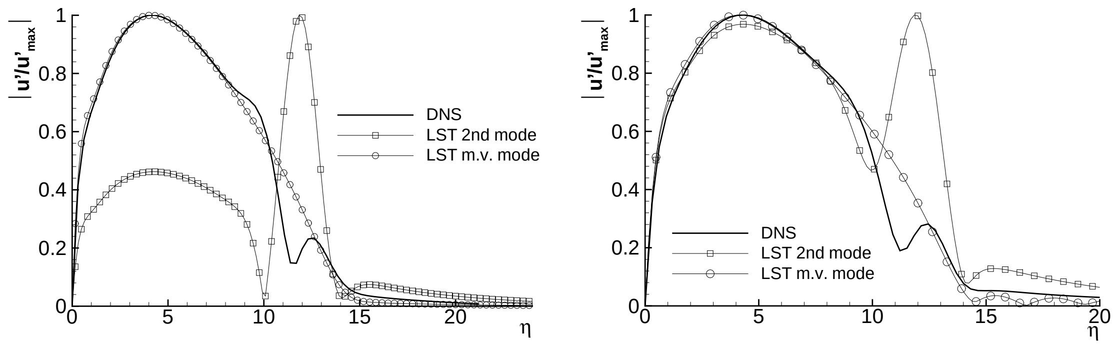


(a) flat plate

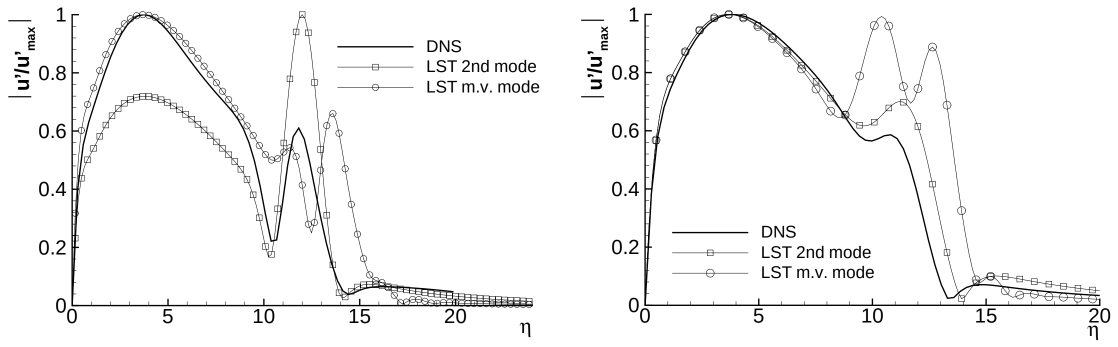
(b) cone

Figure 6: Phase speed $c_{ph,x}$ of primary and secondary wave

and the disturbance amplitudes are identical). The phase speeds of (1,0) and (1,1) for flat plate and cone are shown in Fig. 6. In both cases we at first notice that the phase speed of (1,1) is greater than 1 for some distance, which typically is not the case for a 1st or 2nd-mode disturbance. It turns out that the disturbance strip also excites another 3-d mode which can also be found in LST, a so-called multiple-viscous-solutions (m.v.s.) mode. The phase speed of this mode according to LST is also included in Fig. 6. The receptivity of the m.v.s. mode is much greater than that of the 3-d 2nd mode, so it dominates (1,1) at the beginning as we can see from a comparison of the u'-amplitude distribution from DNS and the eigenfunctions from LST in Fig. 7 a) and b). Since the m.v.s. mode is more damped than the 3-d 2nd mode the distribution gradually moves over to the 2nd mode (see Fig. 7 c) and d). In the case of the cone this happens much faster because, due to the decreasing propagation angle, the 3-d 2nd mode is even amplified from a certain point ($x \approx 52$ according to LST; at exactly this position (1,$\pm$1) stops decaying) as it gets more and more similar to the unstable 2-d mode. The 3-d 2nd mode is much more prone to go into resonance than the m.v.s. mode because the phase speed is closer to that of the 2-d primary wave. This holds even more for the cone because the phase speeds of 2-d and 3-d 2nd-mode waves again tend to be the same going downstream. This renders the necessary threshold amplitude on the cone to be only half that on



(a) flat plate, $x = 67.6$ (b) cone, $x = 49.5$



(c) flat plate, $x = 77$ (d) cone, $x = 58.8$

Figure 7: Amplitude distributions (DNS) of (1,1) and eigenfunctions (LST)

the flat plate: On the flat plate, resonance starts at $x \approx 81.2$ at a (1,0)-threshold amplitude of 8 %, compared to 4 % on the cone at $x \approx 59.7$.

For the likeliness of the different transition scenarios at flight conditions with wall-cooling we find the following: Oblique-type transition is a robust transition mechanism for both the flat plate and the cone, also at cold wind-tunnel conditions (recall that wall-cooling stabilizes the vorticity modes). Important is the beginning of the unstable region. This is quite early for 3-d vorticity modes, and three times earlier on the flat plate than on the cone. Since relevant 2-d N-factors, decisive for fundamental breakdown, are reached only late on the flat plate, oblique breakdown is expected to be the most relevant mechanism there. On the cone however, sufficiently large 2-d N-factors are reached earlier and the behaviour of the secondary 3-d waves accelerates the fundamental mechanism as described above. Here, fundamental transition caused by 2-d acoustic modes significantly destabilized by the wall-cooling, seems to be the dominant mechanism.

The possibility of the appearance of fundamental breakdown on the cone has an important consequence in practice. It is a mechanism caused by acoustic disturbances which have their maximum located near the wall. Therefore the heating of the wall in the transitional regime is much larger than in the oblique-type scenario. This is verified by the examination of the wall temperatures computed using a radiation-adiabatic wall-temperature condition for the instantaneous total flow. The time-averaged wall-temperature alteration for the fundamental case on the cone is shown in Fig. 8. The heating in the computed area reaches 250 K, which is more than 25 % of the wall temperature of the laminar base flow (≈ 950 K). We note that the heating in the transitional regime is known to be higher than in the turbulent regime.[5]

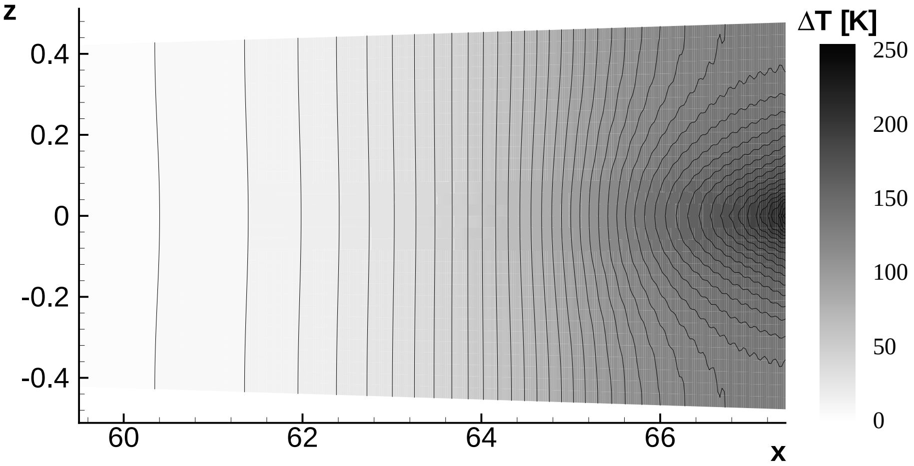


Figure 8: Averaged wall-temperature alteration; cone, fundamental-type transition

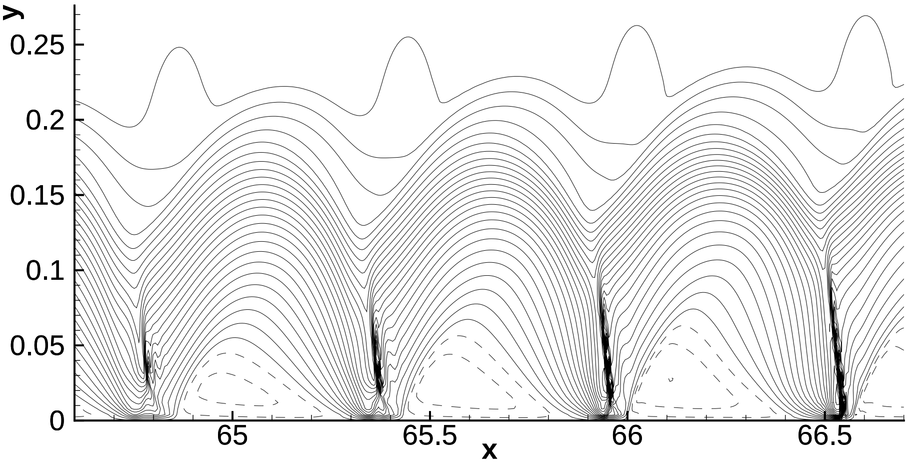


Figure 9: Isocontours of instantaneous velocity u (increment 0.04, negative values dashed); cone, fundamental-type transition

High-amplitude acoustic modes cause complex physical phenomena. These disturbances have the special characteristic that they travel by more than the speed of sound faster than the base flow in a relative supersonic region near the wall. However, this only holds if the amplitudes are small. As the amplitudes increase, we observe the formation of shocklets. They can be clearly seen in Fig. 9 where isolines of u are plotted. Dashed lines correspond to negative values, so strong local flow separation areas are also visible. They are caused by the presence of the strong 2-d wave with its near-wall maximum. The simulation of these phenomena needs an enormous computational effort, and they are the reason why the simulations can not yet be easily extended into the turbulent regime.

5 CONCLUSIONS

From LST it is known that the integral amplification of 2-d 2nd-mode disturbances is higher on the cone. This compensates the fact that the unstable region starts three times later by far, so that fundamental breakdown caused by these modes is strongly favoured on the cone, especially at flight conditions with wall cooling which destabilizes the acoustic modes. DNS results show that fundamental breakdown is even more accelerated compared to the flat plate by the decreasing propagation angle of 3-d waves. This leads to the conjecture that the heating and the shear stresses at the wall will be significantly higher on the cone, because these effects are much more pronounced for fundamental breakdown. On the flat plate oblique breakdown, which is delayed on the cone by the weaker-growing steady longitudinal vortex mode, is expected to be the dominant transition mechanism.

6 ACKNOWLEDGEMENTS

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