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The influence of dissipation laws on the calculation of turbulent boundary layers with pressure rise

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Summary: For calculations of boundary layers by integral methods which use the energy equation a closure-relation for the energy dissipation is needed. The choice of this empirical law can have strong influence on the result especially on boundary layers with pressure gradients. This is demonstrated by calculations of layers with a constant shape factor which are approximately so-called equilibrium layers. Their velocity distributions are calculated by an integral method using different C_D-relations and with empirical formulas for equilibrium layers for comparison.

Der Einfluß von Dissipationsgesetzen bei der Berechnung turbulenter Grenzschichten mit Druckanstieg

Übersicht: Für Grenzschichtrechnungen mit Integralverfahren, die den Energiesatz benützen, wird eine Beziehung für die Energiedissipation benötigt, welche nur empirisch gewonnen werden kann. Die Wahl des Dissipationsgesetzes kann, vor allem bei Grenzschichten mit größerem Druckanstieg, das Ergebnis stark beeinflussen. Dies wird anhand von Grenzschichten mit konstantem Formparameter gezeigt. Geschwindigkeitsverteilungen für solche, auch Äquilibriumsgrenzschichten genannte, Reibungsschichten werden mit einem Integralverfahren mit verschiedenen C_D-Beziehungen und zum Vergleich nach empirischen Äquilibriumsbeziehungen berechnet.

List of symbols

U Velocity normalized with free-stream velocity U_∞

X distance along chord normalized with chord length c

Re Reynolds number $= \dfrac{U_\infty c}{\nu}$

$Re_{\delta 2}$ Reynolds number $= \dfrac{U_\delta \delta_2}{\nu}$

c_D Non-dimensional dissipation integral $= \dfrac{1}{\frac{\varrho}{2} U_\delta^3} \int_0^\infty \tau \dfrac{\delta U}{\delta y} \mathrm{d}y$

c_F Non-dimensional skin friction coefficient $= \dfrac{\tau_W}{\frac{\varrho}{2} U_\delta^2}$

τ_W Wall shear stress

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δ Boundary layer thickness

δ_1 Displacement thickness $= \int_0^\delta \left(1 - \dfrac{U}{U_\delta}\right) \mathrm{d}y$

δ_2 Momentum thickness $= \int_0^\delta \dfrac{U}{U_\delta}\left(1 - \dfrac{U}{U_\delta}\right) \mathrm{d}y$

δ_3 Energy thickness $= \int_0^\delta \dfrac{U}{U_\delta}\left(1 - \left(\dfrac{U}{U_\delta}\right)^2\right) \mathrm{d}y$

H_{12} Shape factor $= \delta_1/\delta_2$

H_{32} Shape factor $= \delta_3/\delta_2$

I Shape factor $= \dfrac{H_{12}-1}{H_{12}} \cdot \dfrac{1}{\sqrt{c_f/2}}$

Π Equilibrium Parameter $= \dfrac{\mathrm{d}p}{\mathrm{d}x} \cdot \dfrac{\delta_1}{\tau_W} = -\dfrac{\delta_1 \dfrac{\mathrm{d}U_\delta}{\mathrm{d}x}}{c_f/2 \cdot U_\delta}$

1 Introduction

On airfoils the pressure recovery on the suction side is essential in achieving low drag as well as high lift. Wortmann startet that a concave pressure distribution with continuously decreasing gradients has a favourable effect on the development of the turbulent boundary layer [1]. Smith [2], Liebeck [3, 4] and Ormsbee [4] designed airfoils with high lift using the so-called Stratford distribution [5]. This is a distribution which just avoids separation along its entire length, thus allowing the largest possible pressure recovery. Stratford [6] checked this flow experimentally.

In his book on airfoil design Eppler [7, 14] calculated the maximum possible pressure rise for constant shape factors H_{32} with his boundary layer code which is based on an integral method. Starting with the nondimensional velocity $U(0) = 1.0$ at the beginning of the pressure rise he found that the velocity $U(1)$ at the trailing edge decreases with decreasing H_{32} having a minimum for $H_{32} \approx 1.62$, and then increases again when H_{32} still continues to decrease. On the other hand the momentum thickness δ_2 reaches a maximum for $H_{32} = 1.62$. In comparison to Stratford's experiment which yields $U(1) = 0.59$, Epplers result is $U(1) = 0.77$.

In spite of modifications of the empirical functions used in his integral-method he did not succeed in getting a better agreement with Stratfords experiment. As this result is not understandable from a physical point of view comparable

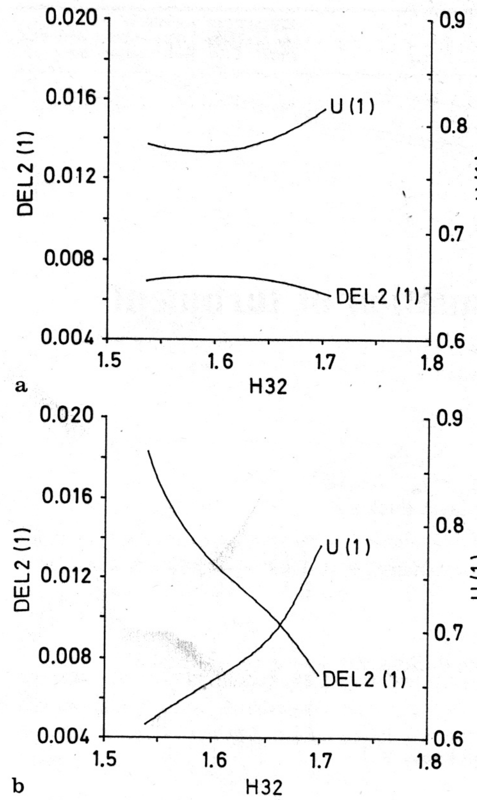

Fig. 1 a, b. Velocity $U(1)$ and momentum thickness $\delta_2(1)$ at $X = 1$ for H_{32} = constant. **a** C_D according to Eppler [7]; **b** C_D according to Felsch [9]

calculations were done with an integral-method according to Geropp [8].

2 Calculation of maximum pressure-rise with different dissipation-laws

Given the shape factor H_{32} = constant the corresponding velocity $U(X)$ was calculated by iteration. For the closure conditions H_{12} (H_{32}) and C_f (H_{12}, $Re_{\delta 2}$) empirical relations according to Felsch [9] were used. Different relations were used for the dissipation coefficient C_D:

a) C_D according to Eppler [7]

$$C_D = 0.01\,[(H_{12} - 1) \cdot Re_{\delta 2}]^{-1/6}$$

In conformity with Eppler $U(1) = 0.77$ results as a minimum and δ_2 shows a maximum for $H_{32} = 1.62$ (Fig. 1 a).

b) C_D according to Felsch [9, 10]:
In this case the velocity $U(1)$ at the trailing edge steadily decreases with decreasing H_{32} while the momentum thickness δ_2 (1) is growing (Fig. 1 b). For $H_{32} = 1.54$ the trailing edge velocity is $U(1) = 0.61$ being not far from $U(1) = 0.59$ according to Stratford. Felsch's C_D-relation is composed of several empirical functions.

Using a C_D-relation for equilibrium boundary layers from Drela [11]

$$C_D = \frac{c_f}{2} \cdot \left(\frac{4}{H_{12}} - 1\right) \cdot \frac{1}{3} + 0.03 \cdot \left(\frac{H_{12} - 1}{H_{12}}\right)^3$$

yields a similar result as b), also showing no minimum for $U(1)$. C_D according to Escudier-Nicoll [10] shows nearly identical results.

If all empirical closure relations are used as in [7] the same results for $U(1)$ and $\delta_2(1)$ are obtained. This means that the dissipation law used in [7] is the reason for the peculiar results. With the other C_D relations cited, velocities at the trailing edge similar to Stratford are reached, only his pressure gradient at the beginning is steeper.

3 Graphical presentation of dissipation-laws

Figure 2 shows a graph of the C_D-relation according to a) over H_{32} and $Re_{\delta 2}$ as a parameter. C_D increases with increasing H_{32}. A graph based on the relation of Escudier-Nicoll (similar to Drela's) is drawn as a dashed line on the same figure. Here C_D increases with decreasing H_{32} approaching separation where it is more than two times larger. Only near the Blasius flow ($H_{32} = 1.73$) both C_D are nearly equal. The influence of $Re_{\delta 2}$ is small.

As boundary layers with constant H_{32} are approximately equilibrium layers and most of the boundary layers on the suction side of airfoils are not far from equilibrium, C_D relations for equilibrium layers must be used. The C_D-coefficient due to Felsch [9] depends on several parameters which account for the departure from equilibrium.

4 Calculated pressure distributions in comparison with experiments

The boundary conditions for Stratford's experiment are $Re = 1.0 \cdot 10^6$ and $Re_{\delta 2} = 2271$ [7]. Starting with $U(0) = 1.0$ at $X = 0$ velocity distributions were calculated for different shape factors H_{32} using the integral method with the C_D relation due to Felsch in the iterative manner as mentioned above. The graphs in Fig. 3 show some distributions. When

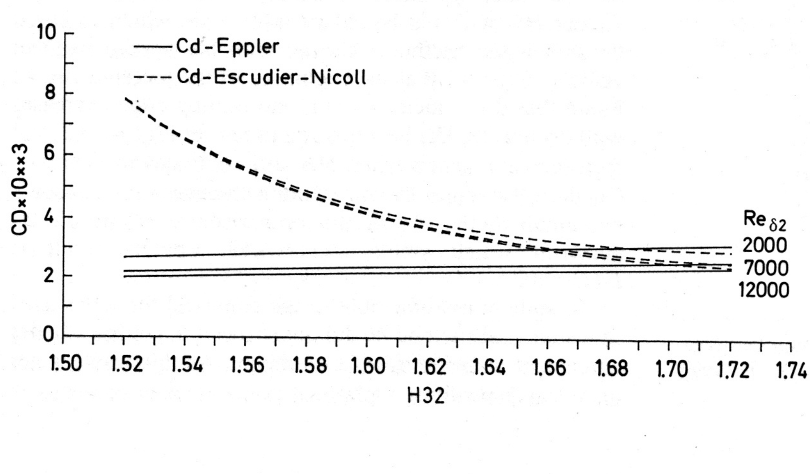

Fig. 2. Dissipation coefficient C_D in relation to H_{32} and $Re_{\delta 2}$ according to Eppler [7] and Escudier-Nicoll [10]

H_{32} approaches separation the differences in U become smaller. The curves for $H_{32} = 1.54$ and $H_{32} = 1.53$ can be distinguished only at the very beginning. They show the lowest velocity $U(1)$ at the trailing edge achievable by this method. On Fig. 4 a distribution with $H_{32} = 1.544$ ($H_{12} = 2.17$) is compared with Stratford's experiment. At the start of the pressure rise Stratford's velocity gradient is steeper, his final velocity is somewhat smaller. With consideration of the approximating calculation the result is sufficient.

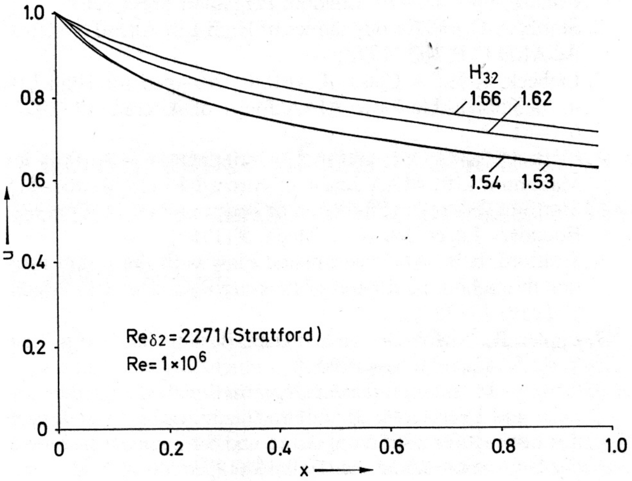

Fig. 3. Velocity distribution $U(X)$ for different shape factors H_{32} = constant with boundary conditions of Stratford's experiment

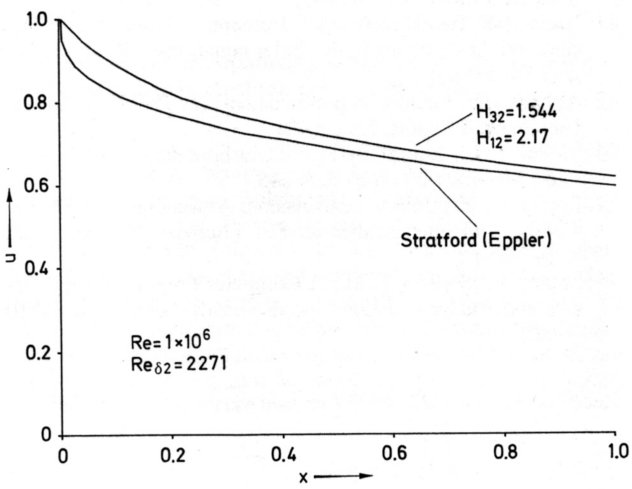

Fig. 4. Velocity distribution $U(X)$ for $H_{32} = 1.544$ near separation and Stratford's experimental distribution

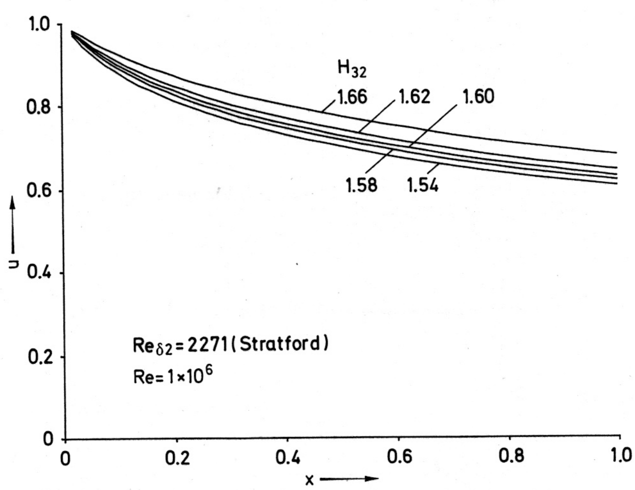

Fig. 5. Velocity distribution $U \sim X^m$ for equilibrium layers with constant shape factors H_{32}

Eppler cites experiments of the present author on a symmetrical airfoil with 34.5% thickness [12]. For a test at an angle of attack of 3° the calculation of the boundary layer with his C_D-relation shows separation at $X/t = 0.6$, while on the contrary calculations with the other C_D-relations and the experiment yield none.

Due to the present calculations the velocity at the trailing edge $U(1)$ for $H_{32} = 1.62$ is 13% lower. As the momentum thickness $\delta_2(1)$ is higher, the drag calculated by the Squire and Young formula is 3% higher. For $H_{32} = 1.54$ the trailing edge velocity $U(1)$ is 20% less than in [7]. The considerations in [7] with respect to the maximum pressure rise and the optimum distribution of the shape factor $H_{32}(X)$ for airfoil design are not applicable. The differences are caused by the insufficient relation for C_D used in [7] and in the airfoil design code [15]. Concerning the design of airfoils, maximum lift could be higher, they could be thicker or the region of turbulent pressure rise could be shorter allowing a longer laminar run.

5 Pressure distributions calculated with relations for equilibrium boundary layers

The velocity distributions for equilibrium boundary layers are of the form $U \sim x^m$. They can be calculated for constant H_{32} without the use of C_D-laws. The shape factor Π can be interpreted as the relation of the pressure forces to the forces due to viscosity. For equilibrium boundary layers $\Pi(x) = \text{const.}$

An empirical relation [13] exists between the shape factors Π and I while

$$\Pi = \frac{(I + 1.5)^2}{36} - 1.8 \text{ and } I = \frac{H_{12} - 1}{H_{12}} \cdot \frac{1}{\sqrt{c_f/2}}.$$

With H_{12} and $Re_{\delta 2}$ given, the wall shear stress C_f can be calculated from empirical relations (f.i. Ludwieg-Tillmann). So I and Π are known.

Velocity distributions for equilibrium layers are given by

$$U = (1 + A \cdot \Delta x)^m.$$

Mellor and Gibson [13] show that

$$1/m = -1 - \left[H_{12} \left(1 + \frac{1}{\Pi} \right) + 1 \right] \left[\frac{1 + (\gamma/\varkappa)}{1 + (\gamma/\varkappa) \cdot (H_{12} - 1)} \right]$$

with $\varkappa = 0.41$ and $\gamma = \sqrt{(c_f/2)}$ and

$$A = -\frac{\Pi \cdot \gamma_0^2}{\delta_{10} \cdot m}.$$

The index 0 marks the values at the start and $\Delta x = x - x_0$. The displacement thickness δ_1

$$\frac{\delta_1}{\delta_{10}} = 1 + A \cdot \Delta x$$

grows linearly with Δx for these boundary layers.

By this way velocity distributions for equilibrium layers can be calculated without empirical relations for the dissipation coefficient C_D. Figure 5 shows some distributions for

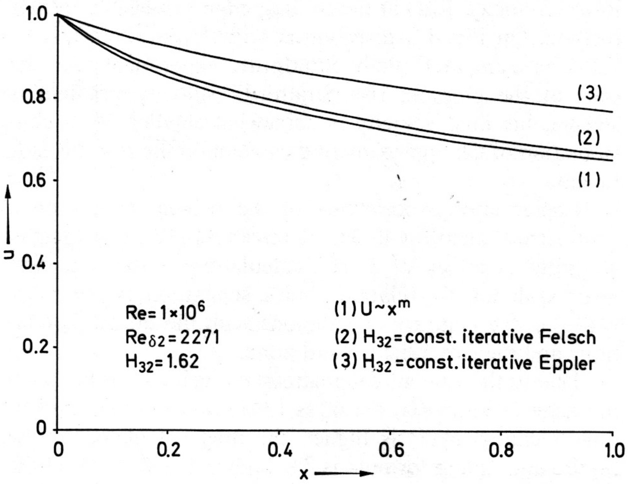

Fig. 6. Velocity distributions $U(X)$ calculated with different empirical relations for $H_{32} = 1.62$ with Stratford's boundary conditions

the conditions of Stratford's experiment with different shape factors H_{32}. A comparison with Fig. 3 reveals, that the distributions for $H_{32} = 1.54$ (near separation) are nearly identical.

6 Comparison of pressure distributions calculated by the integral method and equilibrium relations

In Fig. 6 velocity distributions for the same shape factor $H_{32} = 1.62$ calculated in different ways are compared. Distribution (1) with $U \sim x^m$ gives the largest pressure rise. The iterative calculation for constant H_{32} with closure relations due to Felsch [9], distribution (2) is very similar to (1). Distribution (3) was calculated in the same way, but with closure relations due to Eppler [7]. Owing to the empirical C_D relation its velocity at the trailing edge is much higher.

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