

A critical review of the physical aspects of airfoil design  
at low Mach numbers

by

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In the first two decades of aviation, airfoils were simply copies of the patterns which Nature demonstrates in the variety of birds. Later on, with the advent of monoplanes, thicker airfoils were needed, and at this time man started a long and rarely interrupted quest for better airfoils for a large variety of purposes. This led to large and expensive experimental research activities in the third and fourth decades of our century. The accumulated experience resulted in airfoil catalogues from which a user such as an aircraft designer could make his choice. Today the state of the art is different: At the beginning a more or less defined set of the desired airfoil qualities and the conditions under which these qualities should be performed are gathered and the airfoil designer must find out what type of airfoil would meet these conditions best. Very often there are conflicting requirements, and it is not easy to ascertain the benefits of different compromises. Even the type of wing construction is one important part of the boundary conditions and aircraft designers are sometimes not aware of the strong reciprocal effect between airfoil design and wing construction. Therefore it might be useful to sketch some guidelines which enable the airfoil designer as well as the user to find better airfoils.

I. Friction drag

At low angles of attack and Mach numbers below the critical Mach number, the friction drag is the overriding factor in the profile drag. It is primarily the Reynolds number which characterizes the flow environment. In this paper the Reynolds number may vary between .4 to 40 millions. At lower Reynolds numbers the airfoil flow is complicated by the increasing danger of separa-

tion and the higher Reynolds numbers are usually combined with critical Mach numbers.

a) Basic considerations

1) The first and most effective way to reduce friction avoids a turbulent boundary layer as long as possible. This is best illuminated by comparing the laminar and the turbulent boundary skin friction for a flat plate with different positions of transition. It can be seen from Fig.1 that the drag is relatively more influenced when the Reynolds number is larger and/or when the transition point moves backwards. This picture can roughly be translated into airfoil drag by multiplying the flat plate values by  $1+2\delta$ , where  $\delta$  is the related airfoil thickness.

The all important position of transition is governed by two parameters: the stability of the laminar boundary layer and the perturbations introduced into the laminar boundary layer. The stability in turn depends on one hand on the Reynolds number, and on the other hand on the pressure gradient in flow direction. At the upper end of the Reynolds number range mentioned above, the stability seems to fade out, whereas at the lower end it becomes difficult to overcome the stability and to provoke turbulence at just the right moment.

Perturbations are fed into the boundary layer by surface waviness and roughness. The latter may be caused by connecting skin steps, access openings, leakage, insects, erosion or the turbulence of the free stream.

2) There is always an adverse pressure gradient in the aft part of airfoils, and since our present state of technology yields no hope for a complete laminarization by suction, we have to live with turbulent boundary layers. The second principle of drag reduction therefore takes care that the turbulent boundary layer develops in a favorable manner. Today it is very well known that a "concave" pressure or velocity distribution has advantages, and this has been applied to airfoils, very often successfully. Fig.2 shows an early and typical example of how different velocity distributions influ-

ence the boundary layer thickness at the trailing edge [1]. The gain in drag reduction is of the same order of magnitude as the airfoil thickness and becomes more pronounced with increasing Reynolds number. However, some caution is necessary because most airfoils have to work also under off-design conditions.

3) Between the laminar and the turbulent part of the boundary layer lies the transition region. Very often it is quite important to control this region carefully and to avoid, if possible, laminar separation bubbles, which may spoil the initial conditions of the turbulent boundary layer. This is especially true when a concave pressure distribution follows the transition region [2]. The solution for the transition region is a small range with a slight adverse pressure gradient which destabilizes the laminar boundary layer without separation. It may be called instability range.

With those three basic principles of boundary layer control in mind we can start to solve a first simple design task: to find a symmetrical airfoil with the lowest drag at zero angle of attack for a certain given airfoil thickness and Reynolds number.

b) The symmetrical airfoil with zero angle of attack

As long as the surface curvature is small in proportion to the boundary layer thickness, the geometry of an airfoil has only a minor influence on the boundary layer development which is then determined only by the pressure or velocity distribution. In such cases the form of the airfoil can be considered purely as a medium to produce a certain velocity distribution, and for symmetrical airfoils at low angles of attack this is mostly true. Therefore it is reasonable to start the quest with an educated guess for a velocity distribution. This however produces another problem since not every velocity distribution can be realized by a real airfoil. Obviously it is not possible to solve even this simple task directly but one has to rely on iterative steps.

Fig.3 gives an example: the Reynolds number may be  $4 \cdot 10^6$ . The stability of the laminar boundary layer is just large enough to certify the laminar state by a constant velocity up to 50-60% of the chord. Behind this point an instability range of 5 to 10% chord length may be necessary to provoke transition and to develop a fully turbulent boundary layer. Downstream of the transition region the velocity has to decrease with stronger gradients in order to close the airfoil with sensible trailing edge angles. This type of velocity distribution may be modified by shifting the transition region, say backwards. Under the constraint of a constant airfoil thickness, the supervelocity of the laminar part decreases slightly, and the velocity gradients in the turbulent part increase strongly. The longer laminar part reduces the friction drag, but this gain is very soon overbalanced by the pressure drag due to the faster growing turbulent boundary layer thickness. There exists for a certain Reynolds number a flat optimum for the position of the transition which in turn depends on the prescribed airfoil thickness.

In Fig.4 are shown two velocity distributions for the same goal, minimum drag at zero angle of attack, but the design Reynolds number is ten times higher and lower than in Fig.3. The differences are due mainly to different stability characteristics in the laminar part: at lower Reynolds number even a negative velocity gradient and an expanded transition region are necessary, whereas the contrary is true for the higher Reynolds number.

c) Symmetrical airfoil with low drag bucket

In Fig.3 any angle of attack induces in the nose region on one side a velocity peak with negative, and on the other side with positive velocity gradient. The latter stabilizes the laminar boundary layer more than necessary, and on the other side transition jumps forward, causing a sharp increased drag. At that Reynolds number this airfoil has only a very small lift range with low drag. In order to produce more practical airfoils with a low drag in a wider lift range, we have to



superimpose a positive velocity gradient upon the distributions in Figs.3 and 4, to counterbalance the effect of incidence. It follows that for a constant airfoil thickness, the maximum velocity goes up, as well as the velocity gradients in the turbulent part. A certain drag increase is unavoidable. The ability of the airfoil thickness to counteract the effects of incidence is restricted and for larger low drag ranges it is necessary to shift the transition region forward in order to produce stronger velocity gradients at zero angle of attack. This in turn increases the drag further. Fig.5 gives an example of this relationship for airfoils with different drag buckets. The envelope in Fig.5 is to some extent not even restricted to a certain airfoil thickness. It has a more general significance: suppose, the velocity distributions are all carefully selected to meet the boundary layer principles of the basic considerations. Then for a single-element airfoil with rigid surfaces there seems to exist no further possibility to cross the envelope in Fig.5 to the left. In other words, this envelope seems to be an absolute boundary which cannot be improved.

Sometimes the lowest drag at one single Reynolds number is not the primary goal, but a low drag in a certain range of Reynolds numbers. Usually a reduced sensitiveness of the Reynolds number influence can be achieved by a more extended instability range. Now the position of the transition can easily move for and aft and compensate partly for the adverse effects of Reynolds number on the friction coefficients.

d) Symmetrical airfoils with flaps

If an airfoil can be equipped with flaps, it has a variable camber, and this fact can be used to exceed the drag lift envelope of Fig.5. Let us assume an airfoil as in Fig.3 with practically no low drag range. Basically, the deflected flap causes an aftloading or higher and lower velocities in the environment of the flap. Due to a certain incidence, similar split up velocities can be produced in the front part of the airfoil. Both the flap deflection and the incidence can

act together in producing lift without changing the type of velocity distribution and therefore the drag. In order to exploit the full benefits of this idea we have to remember that a flat plate with a kink at the flap hinge produces at constant angle of attack a velocity peak at the leading edge and at the kink. To eliminate these unfavorable peaks we can use the thickness distribution. In terms of airfoil design this poses a mixed problem in which the flap deflection, i.e. a part of the airfoil geometry, is prescribed, and both the thickness distribution in front of the airfoil and the angle of attack are modified to produce the desired velocity distribution. Some experience with airfoils of 12-15% thickness shows that this problem has reasonable solutions for flap chords of 20 to 30% and prescribed flap angles of 8-12 degrees, yielding an additional low drag lift range of about  $\pm .4$ . Fig.6 shows an airfoil which was optimized in this sense for a flap deflection of 10 degrees. Fig.7 illustrates the benefits of such advanced airfoils. [3]

e) Airfoils with small camber and low drag bucket

All considerations of symmetrical airfoils with and without flaps can be transferred to lifting airfoils as long as the average position of transition is similar. The simplest way to do this uses a constant velocity difference between lower and upper side which yields the well known NACA meanline with  $Q = 1.0$ . However, this type of camber is not always the right solution. Low pitching moments ask for reduced aft loading and imply higher velocities on the upper side in the front part of the airfoil. The contrary is true when the critical Mach number should increase. Fig.8 shows a systematic variation of lift distribution. [2]

With increasing camber, especially when thicker airfoils are cambered, the fiction of a boundary layer which is only controlled by the velocity distribution holds no longer. The geometry of the airfoil becomes equally important. For the turbulent boundary layer, it is the surface curvature which influences the development more and more. In other words, now both the geometry and the velocity distribution must be coupled when considering the boundary layer development.

The physical reason is quite clear: in a curved potential flow, pressure and centrifugal forces perpendicular to the streamlines are in perfect equilibrium. In the boundary layer flow, the centrifugal forces fade out towards the wall, and the flow becomes dynamically stratified, stable on a convex wall and unstable on a concave wall. On the upper side of a cambered airfoil, the turbulence has to work against the stable stratification, the impulse exchange is reduced and the separation moves forward. Unfortunately there seems to be no boundary layer calculation method which takes these long known effects into account, and no generally valid statements on favorable combinations of velocity and airfoil form can be made.

Hence in contrast to the low cambered airfoils, there is some freedom to speculate how the highest lift at low drag may be realized. Some experience seems to indicate that an unseparated flow is not possible when the camber of the upper airfoil surface itself exceeds 15-17%. If this rough guess is accepted it is clear that a thinner airfoil can produce more lift than a thicker one with the same upper surface.

A good example for a highly cambered airfoil with low drag is shown in Fig.9. This airfoil is the extended version of the variable geometry airfoil designed for the British project "Sigma" [4]. The maximum glide ratio goes to 160 at  $c_L = 1.7$  and  $Re = 1.5 \cdot 10^6$ . On the upper side, the transition occurs between 40-50%. Behind the transition region the curvature soon disappears as the thickness of the turbulent boundary layer increases. The design technique of such an airfoil is quite similar to the case of an airfoil with a prescribed deflected flap mentioned above [5]. However, in this case and even more in the following chapters, the feedback of the thick boundary layer on the pressure distribution has to be included in the iterative design process.

In the context of higher cambered airfoils there exists another interesting feature: in potential flow a camberline can easily produce a lift loading up to the trailing edge, shifting the necessary pressure recovery into the free stream be-

hind the trailing edge. However, in reality the boundary layer changes the "fluid" camberline, and the adverse pressure gradients are shifted in front of the trailing edge. This may cause a separation which in turn modifies the fluid camberline even more. This is another example where the feedback of the boundary layer on the pressure distribution has to be considered seriously.

For fixed wing aircraft different  $c_L$ -values of the wing are coupled with different Reynolds numbers. Therefore the design of the lower and upper airfoil surface should take into account the different curvature as well as the different Reynolds number.

## II. Maximum lift

For low drag airfoils which are squeezed out in order to yield the widest possible low drag bucket, the maximum lift is clearly an "off design" condition. However, when the low drag range requirement is relieved, some freedom is gained to include in the design considerations some high lift control.

For low cambered airfoils at higher angles of attack, the nose form plays an important rôle in the development of the upper side boundary layer. Usually the situation in the first few percent of the nose length is characterized by high velocity peaks followed by a deep slope which separates the laminar flow and spoils the initial conditions of the turbulent boundary layer by the laminar separation bubble. Now ideas similar to those in the low drag case can be applied at high angles of attack: due to the low local Reynolds numbers, the thin laminar boundary layer is quite stable and needs a pronounced instability range to provoke turbulence, ideally without any separation bubble. There is however some experimental evidence that it is not necessary to suppress the separation bubble completely. The effect is nearly the same when the bubble stays very thin. The velocity distribution downstream of the transition region is clearly a concave one, and it is well known that the associated turbulent boundary layer reacts very favorably on improved initial conditions.



Such a boundary layer control which is restricted to the first 5-10% of the chord length can help to increase the maximum lift. Figs. 10 and 11 provide an extreme example, where these ideas have been applied to a symmetrical airfoil regardless of low drag considerations at low angles of attack. [6]

### III. Stall

Very often it is not the maximum lift which is of primary interest but the behaviour of the airfoil at and beyond the maximum lift, i.e. in the partly or completely stalled region. A properly designed airfoil should at least avoid the dangerous "leading edge stall". This is not too difficult, and even a mild form of boundary layer control as shown in Fig.10 will change the type of stall into a "trailing edge stall".

Sometimes airfoil users want a  $c_L(\alpha)$  curve which reaches a maximum and stays there. We can understand the trailing edge stall as the result of two counteracting effects: the increasing angle of attack should raise, and the growing separation will lower, the circulation. If both effects cancel each other a constant  $c_L$  will occur. Obviously there are three important parameters: the change of position of transition and of separation and the size of the separated region or the separation angle. For an airfoil whose upper side is squeezed out in order to give a maximum low drag range, the transition point jumps forward too fast and the  $c_L$  beyond the  $c_{Lmax}$  falls down. If the upper edge of the low drag range is allowed to round off, the movement of velocity peaks and hence the transition can be slowed down. Then the same is more or less true for the separation of the turbulent boundary layer and the  $c_L$  may be unchanged during the stall.

The advantage of such an arrangement is that not only the lift curve becomes smoother but also the drag increase is far less severe than with the squeezed out type of airfoil. A typical example of such behaviour is given in Fig.12. [7]

### Conclusion

Airfoil design is always a matter of more or less direct boundary layer control. To accomplish this goal we obviously need airfoil and boundary layer theory, the availability of computers and programs and finally a suitable windtunnel as tools.

It was the purpose of the paper to show that another quality is equally indispensable: imagination which enables one to carve out of the physical aspects of the problem an advanced airfoil. However, the physical aspects are transparent enough to state that we cannot expect a breakthrough. This is especially true for the low cambered, low angle of attack airfoil. Any advances are slow and hard to achieve as one approaches the physical limits. There exist however numerous details in the "airfoil and boundary layer" field where our present knowledge is open to further refinements, and this raises the hope that further advanced airfoil design may also be possible in the future.

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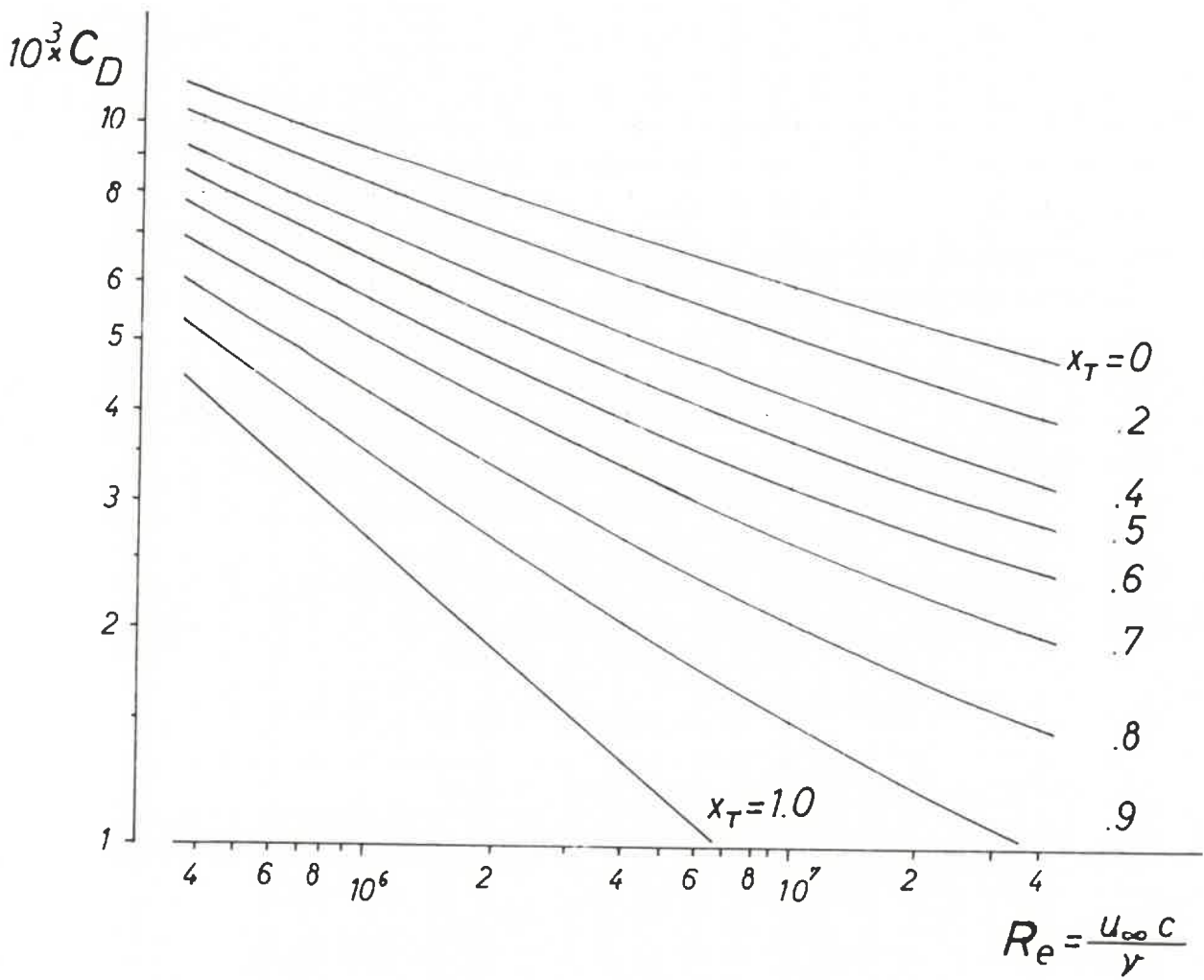


Fig.1 Drag of a flat plate with different position of transition

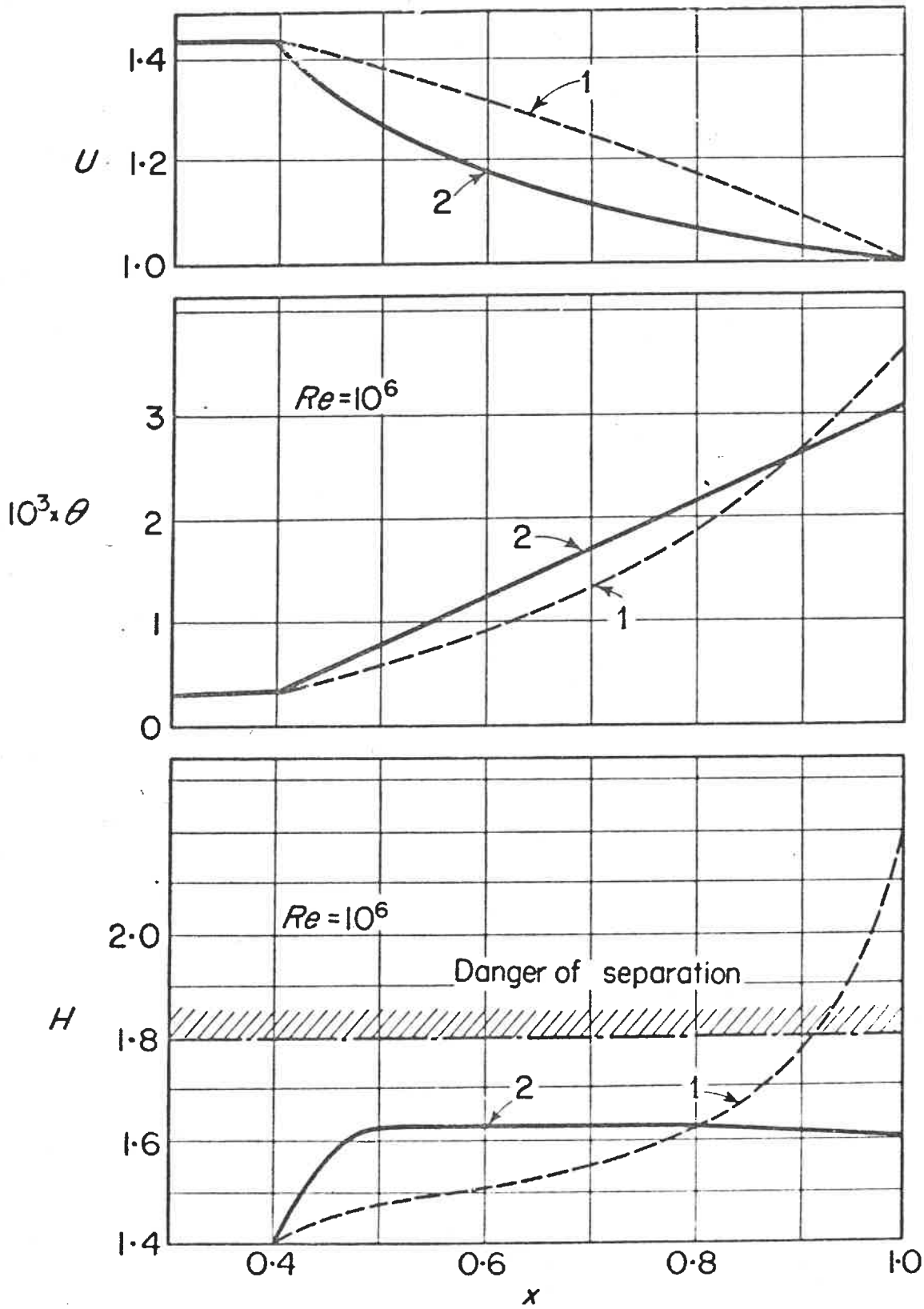


Fig.2 Development of the turbulent boundary layer for two different velocity profiles.  
 $\theta$  = momentum thickness;  $H$  = shape parameter



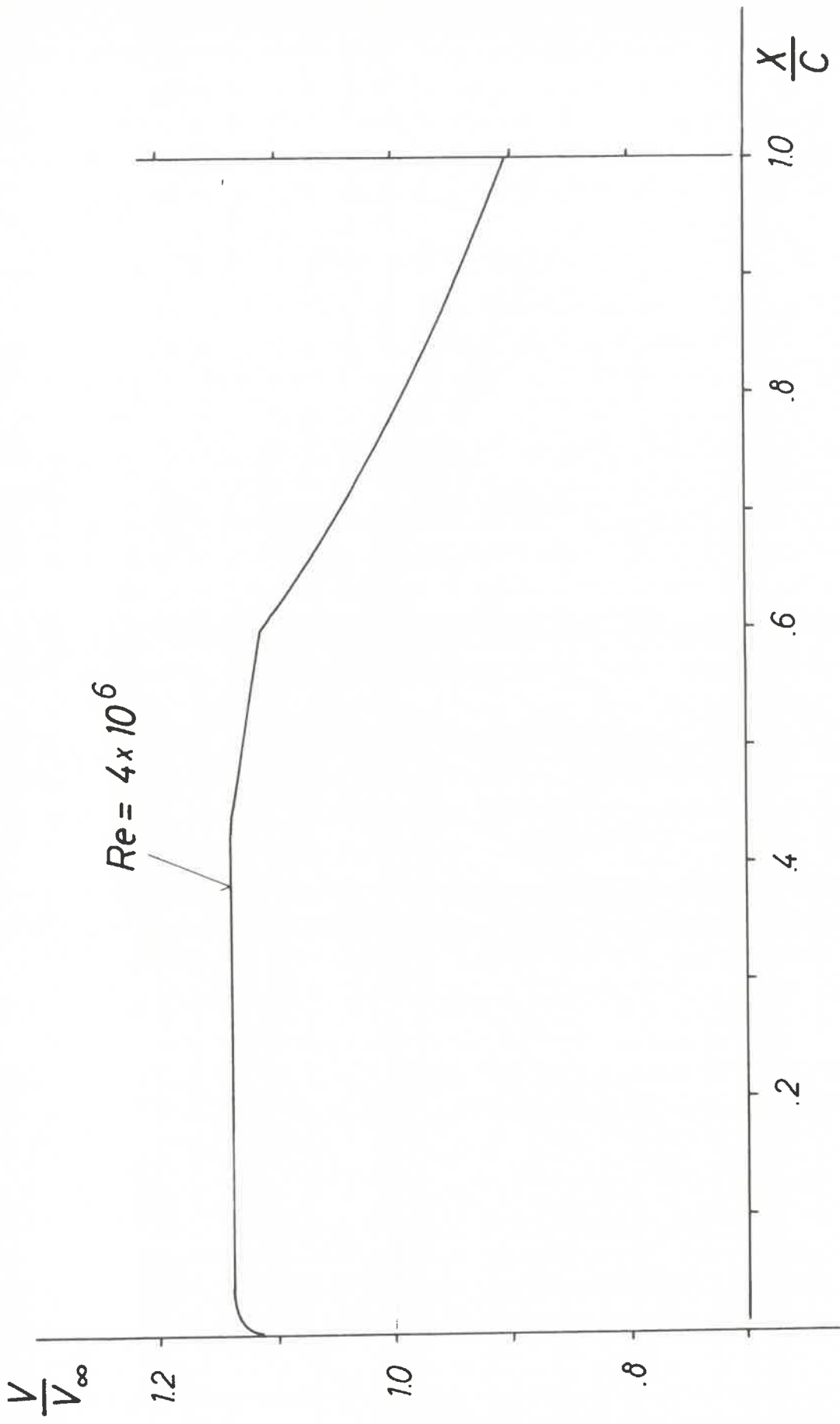


Fig.3 Typical velocity distribution of a symmetrical airfoil optimized for minimum drag at  $Re = 4 \cdot 10^6$

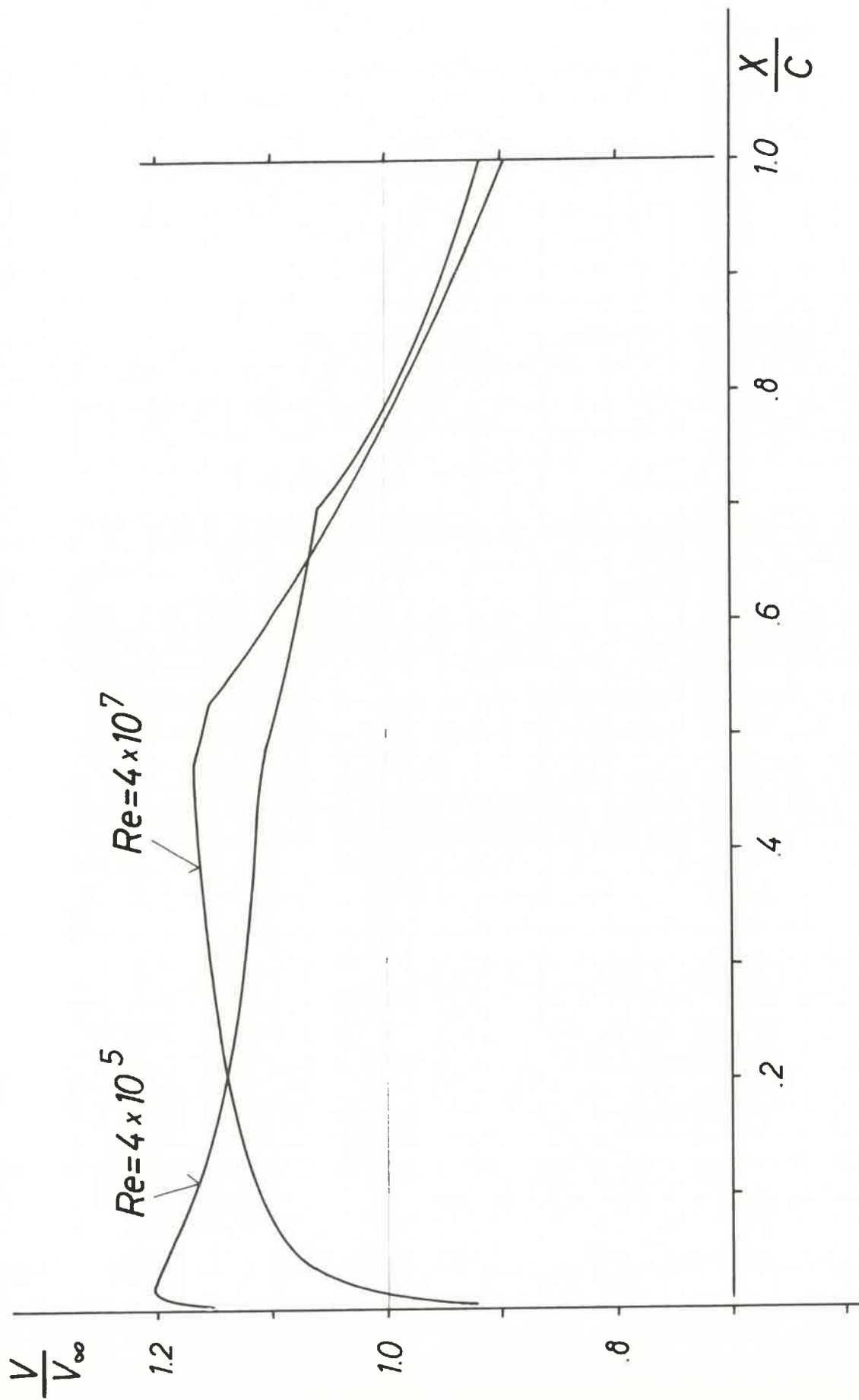


Fig. 4 Typical velocity distribution of two symmetrical airfoils optimized for minimum drag at  $Re = 4.10^5$  and  $4.10^7$

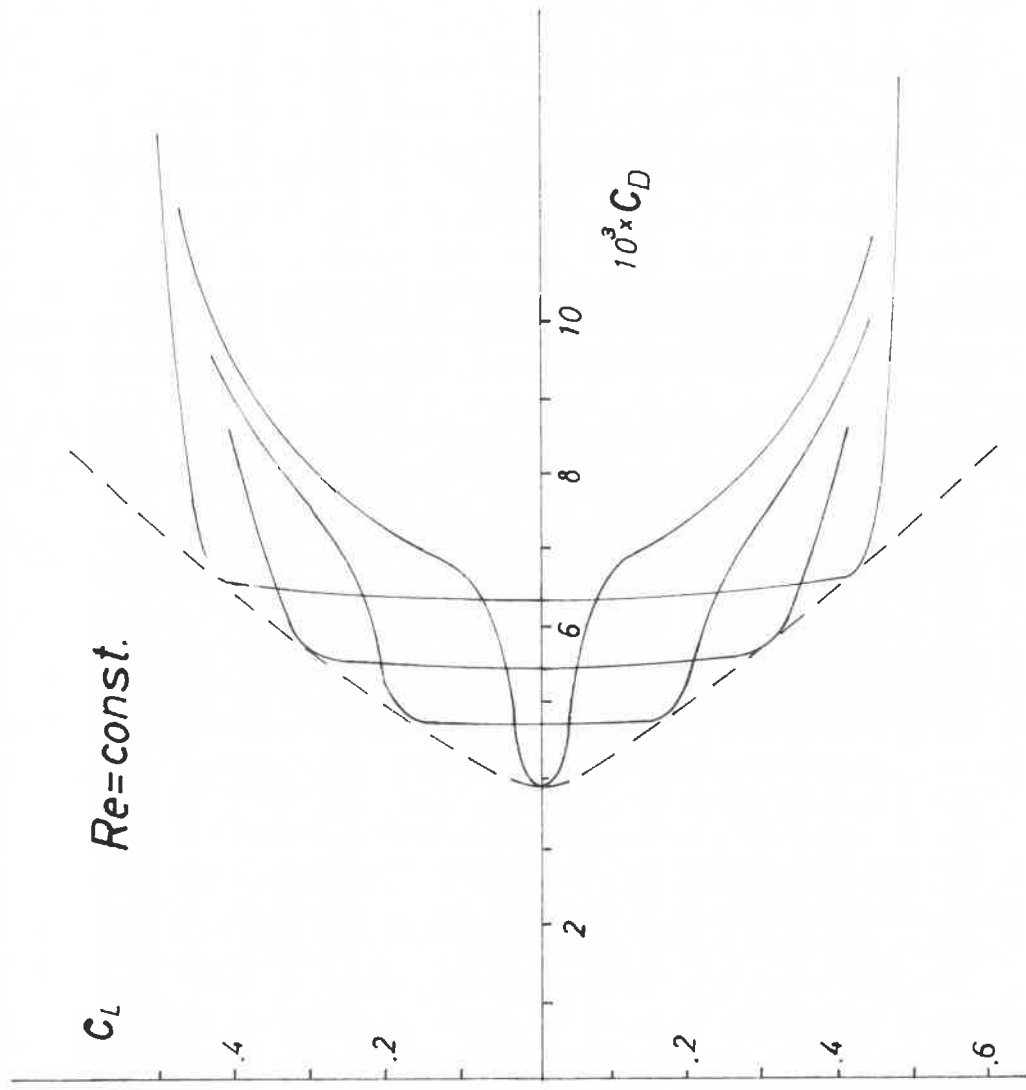
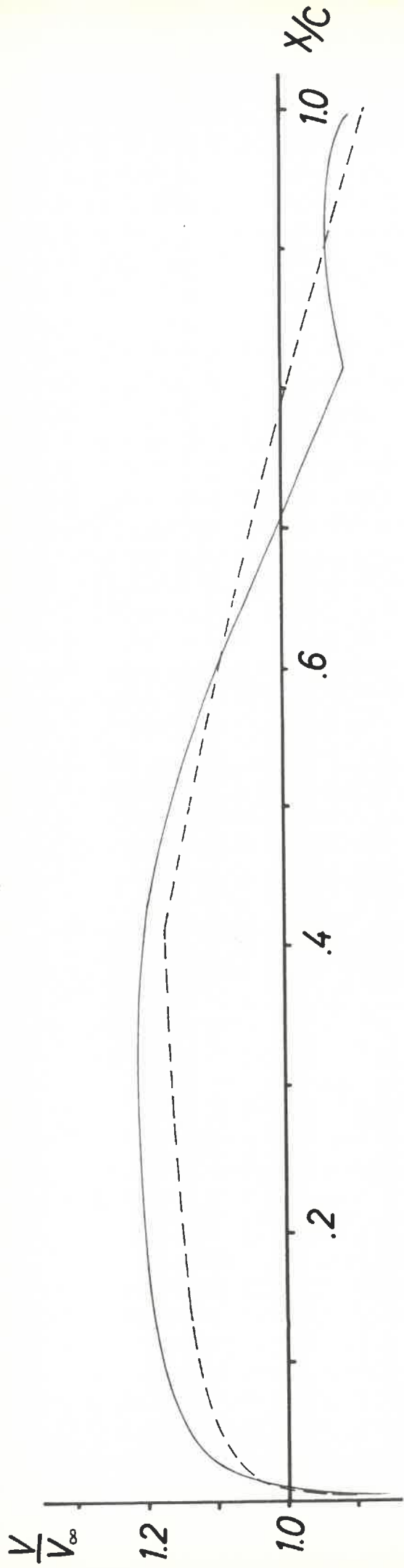


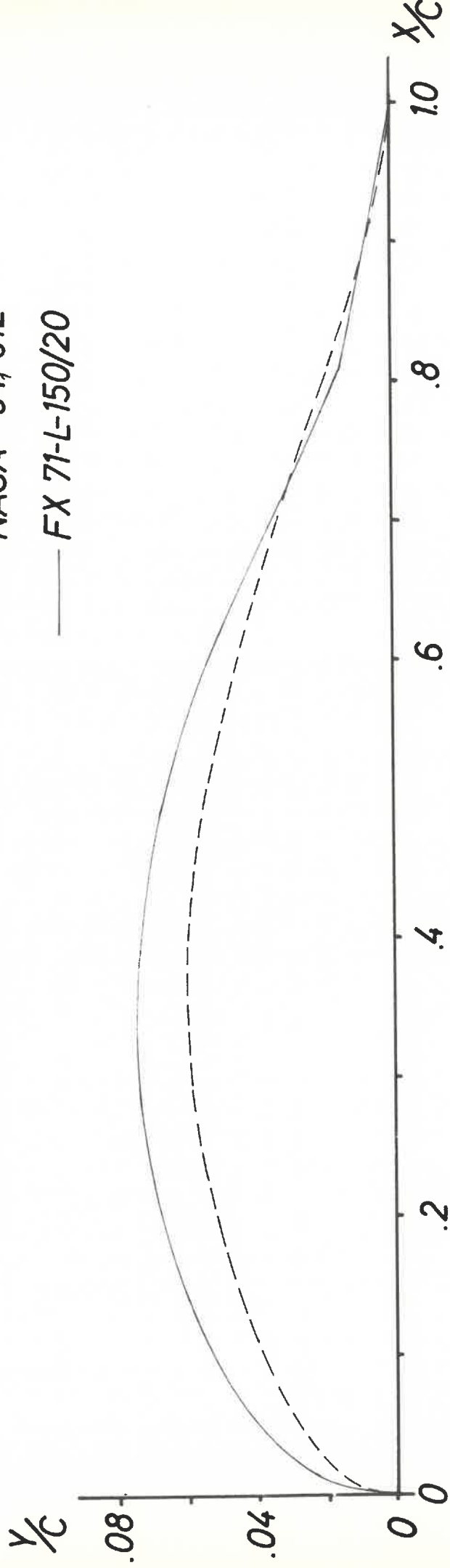
Fig.5 Typical lift-drag polars for different low drag ranges



Velocity distribution

----- NACA 64-012

—— FX 71-L-150/20



Thickness distribution

Fig. 6



FX 61-184

□  $Re = 1.0 \times 10^6$   
△  $Re = 1.5 \times 10^6$   
◻  $Re = 3.0 \times 10^6$

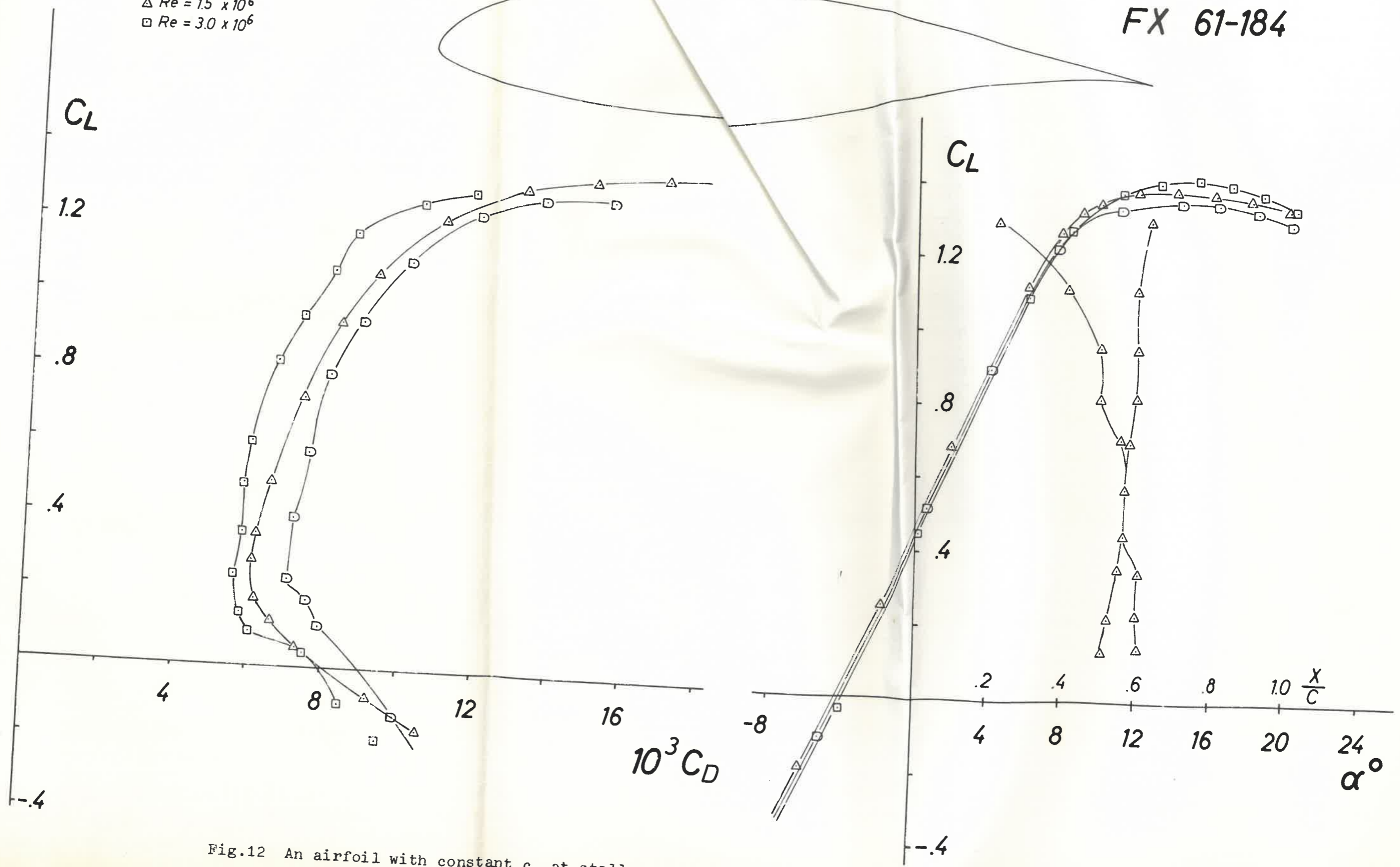


Fig.12 An airfoil with constant  $c_L$  at stall

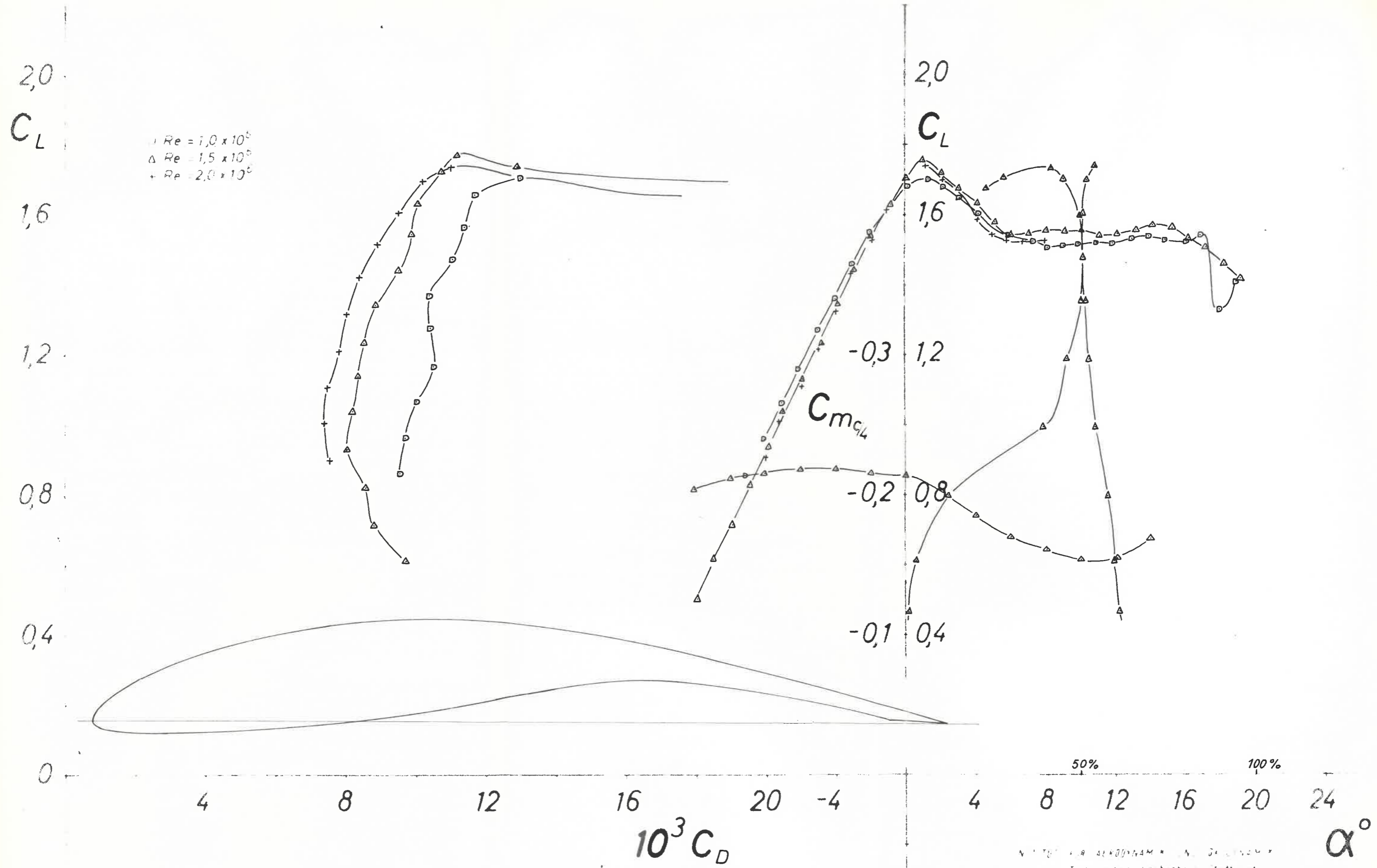


Fig.9 Lift-drag polars,  $c_L(\alpha)$ ,  $c_m(\alpha)$  and  $\frac{x_f}{c}$  of the extended version of the FX 67-VG-170/136 airfoil

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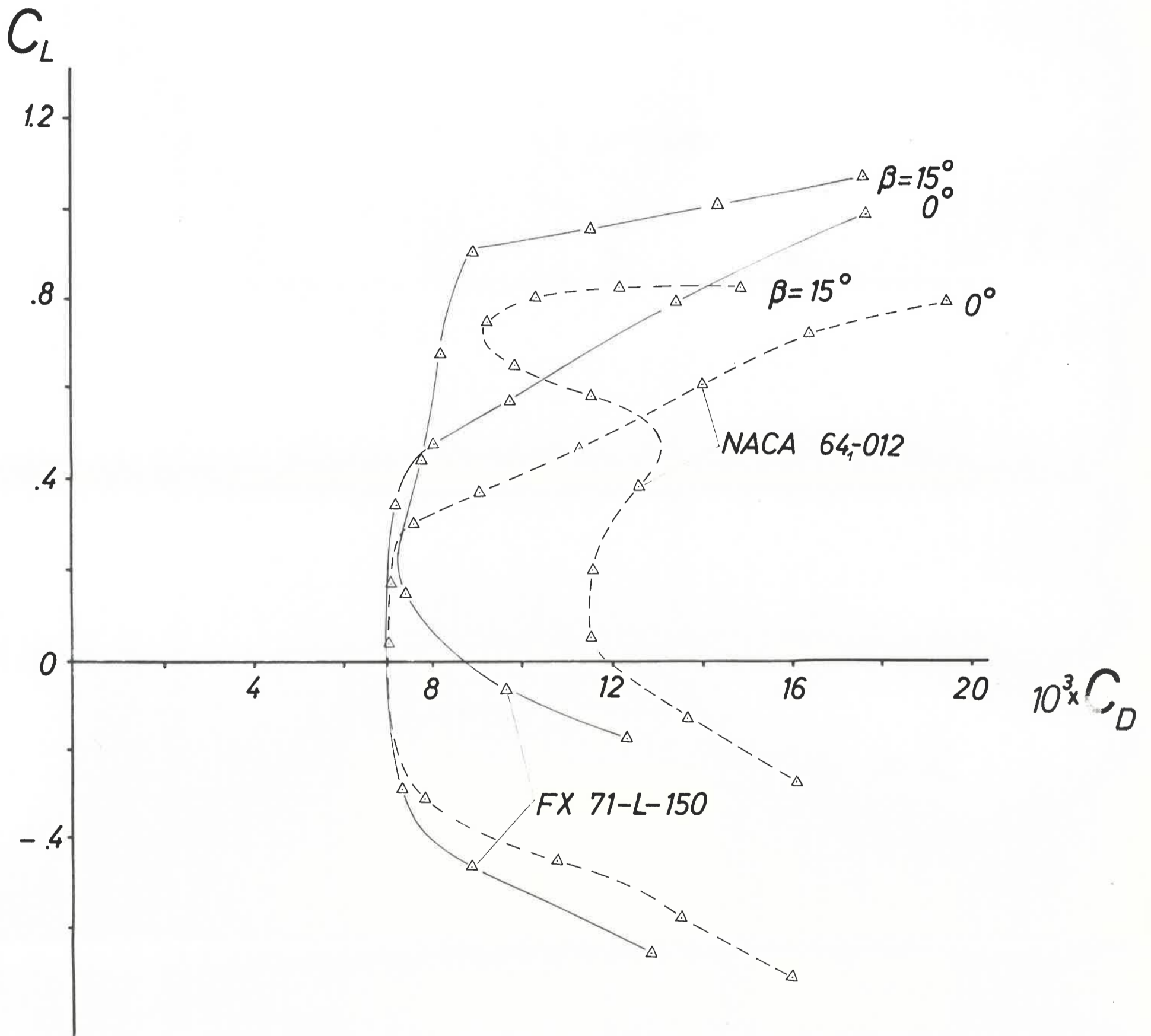


Fig.7 Comparison of an NACA and an optimized airofoil for two flap settings  $Re = 1.10^6$  .