European Congress on Computational Methods in Applied Sciences and Engineering
ECCOMAS 2000
Barcelona, 11-14 September 2000


# STUDY OF THE ACOUSTIC RECEPTIVITY OF A BLASIUS BOUNDARY LAYER IN THE PRESENCE OF A SURFACE NON-UNIFORMITY

**Anke Wörner⋆, Ulrich Rist⋆, Stefan Herr⋆, Werner Würz⋆, Siegfried Wagner⋆ and Yury S. Kachanov†**

⋆Institut für Aero- und Gasdynamik (IAG), Universität Stuttgart,
Pfaffenwaldring 21, 70550 Stuttgart, Germany
e-mail: woerner@iag.uni-stuttgart.de,
web page: http://www.iag.uni-stuttgart.de

†Institute of Theoretical and Applied Mechanics (ITAM), Russian Academy of Science,
630090 Novosibirsk, Russia, e-mail: kachanov@itam.nsc.ru,
web page: http://www.itam.nsc.ru

**Key words:** Receptivity, Laminar-Turbulent Transition, Boundary Layer, Direct Numerical Simulation, Aerodynamics.

**Abstract.** *The generation of 3D Tollmien-Schlichting (TS) waves by the interaction of a plane acoustic wave with a localized surface non-uniformity is studied using direct numerical simulations (DNS). The DNS are based on the vorticity-velocity formulation of the complete Navier-Stokes equations using a uniformly spaced grid in streamwise and wall-normal direction and a spectral representation in spanwise direction. A novel way of modelling the surface non-uniformity is introduced avoiding the use of a body-fitted coordinate system. Acoustic receptivity coefficients for 2D as well as for 3D surface non-uniformities are calculated and compared with those found by means of theoretical approaches and experiments. In all studied cases a very good quantitative agreement between the DNS and the linear receptivity theory as well as the experiments is found.*

## 1 INTRODUCTION

Laminar turbulent transition is a spatially evolving process which can be subdivided into three main stages. In the first stage (called receptivity) external disturbances such as sound waves or vortical disturbances penetrate into the boundary layer where they are tuned to wave-like disturbances. The second stage consists of the linear amplification of these initially created instabilities and the last stage is the nonlinear breakdown to turbulence. If we have a look at this evolution, it gets clear that a correct prediction of the location of transition in many applications requires a correct quantitative prediction of the initial amplitude of the disturbance depending on the amplitude of the external perturbations. Within this framework, direct numerical simulations can be used as a tool for the validation of receptivity theories.

The problem of acoustic roughness receptivity, especially for 3D surface non-uniformities, is only poorly studied yet. There are several theoretical studies concerning linear 2D roughness-acoustic receptivity, basing either on the asymptotic theory (Goldstein 1985 [1]; Ruban 1985 [2]) or the classical Orr-Sommerfeld theory (Zhigulev & Fedorov 1987 [3], Crouch 1992 [5], Choudhari & Streett 1992 [6]). The latter approach accounts for the finite Reynolds-number effects and is therefore often called 'Finite-Reynolds-Number-Theory'. Direct numerical simulations for the 2D case were performed by Kobayashi et al. 1993 [7] and Casalis et al. 1997 [11]. Experimental investigations were carried out by Saric, Hoos & Radetzsky 1991 (2D) [4], Zhou, Liu & Blackwelder 1994 [8], Saric & White 1998 (2D) [12], Würz et al. 1999 [13] and Cullen and Horton 1999 [14].

In this paper the acoustic receptivity of a Blasius boundary layer in the presence of a localized surface non-uniformity is studied using direct numerical simulations. For mutual validation the DNS results are compared with experimental and theoretical ones.

## 2 NUMERICAL METHOD

The DNS are based on the vorticity-velocity formulation of the complete Navier-Stokes equations for incompressible flows using a uniformly spaced grid, fourth-order accurate finite differences in streamwise and wall-normal direction and a spectral representation in spanwise direction. The integration in time is performed with a fourth-order accurate Runge-Kutta scheme. The nonlinear terms in the vorticity transport equations are evaluated pseudo-spectrally, i.e. in physical space, including de-aliasing. The $v$-Poisson equation is solved with a multi-grid method, using a vectorizable, stripe-pattern SOR line-iteration technique on each grid (see Rist & Fasel 1995 [10]). In contrast to Rist & Fasel a 'total variable formulation' is used instead of a 'disturbance variable formulation'. A sketch of the rectangular integration domain is shown in figure 1. The sound wave in the free stream is prescribed at inflow as a solution of the second Stokes' problem.

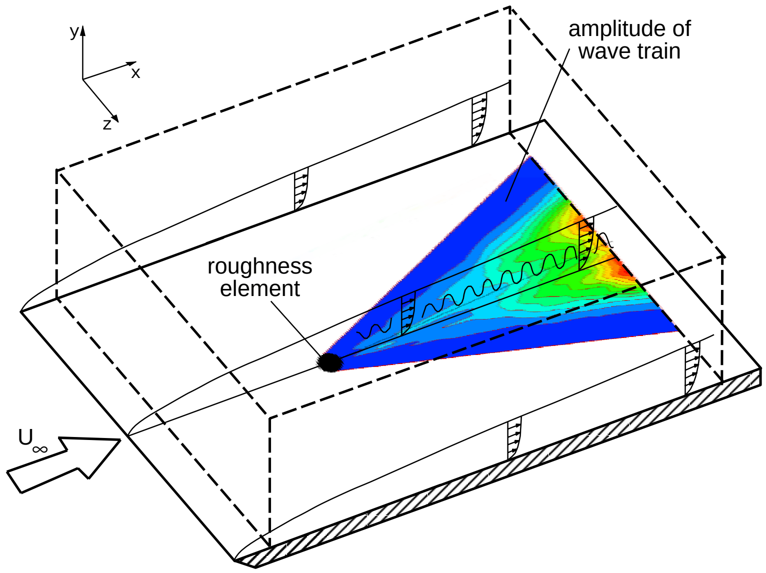


Figure 1: Integration domain

## 2.1 Modelling of the surface roughness

For the modelling of the surface roughness a novel wall model has been implemented. Thereby the use of a body-fitted coordinate system is avoided and the typically small surface roughness is modelled by non-zero velocity at the lowest row of grid points which is extrapolated from the field in such a way as to fulfill the no-slip condition at the surface of the roughness. This is done using fifth-order polynomials which are consistent to the finite-difference representation of the flow field.

The new wall model is compared with standard first-order formulations that consider only a no-slip condition in wall-parallel directions. So the main differences between the new model and these models found in literature are that it takes into account the no-slip condition for the wall-normal velocity as well and that it contains no linearization. Thus, at least for higher surface roughnesses, it should lead to more accurate results.

## 2.2 Receptivity calculations

To study the problem of acoustic roughness receptivity several numerical simulations are needed. In the first simulation the steady flow over a flat plate with a roughness element located at a certain distance from the leading edge is calculated.

In the second simulation the interaction of the sound wave with the roughness element is calculated using the previously calculated steady flow over the roughness as initial condition. The difficulty now is to extract the Tollmien-Schlichting waves which are created by the interaction of the sound wave with the roughness from the total solution. There are two problems. The first one is that the sound wave itself has the same frequency as the created TS-waves and the second one is that there is a numerically created TS-wave starting at the inflow boundary resulting from an approximate inflow boundary condition. This problem is solved using a method suggested by Crouch & Spalart 1995 [9].

Therefore, a third simulation is needed including the sound wave but no roughness. In this simulation the TS-wave created at the inflow boundary is also present. So the TS-wave created at the surface non-uniformity can be extracted from the total solution by subtracting the results of the third simulation from the results of the second one.

## 3 DNS RESULTS

In a 2D simulation a single Tollmien-Schlichting wave can be observed in the boundary layer downstream of the surface roughness. Figure 2 shows the amplitude development of the created disturbance versus $x$, figure 3 the profile of the $u$-velocity of the disturbance versus $y$ at a location downstream of the surface non-uniformity.

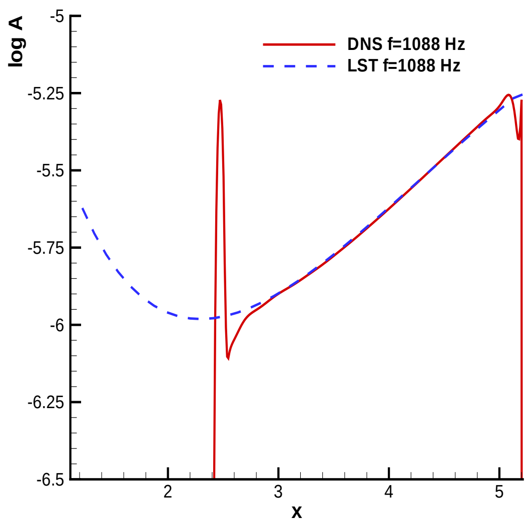


Figure 2: Amplification curve, roughness located at $x = 2.471$, $F = 2\pi f\nu/U_\infty^2 \cdot 10^{-5} = 7.9$

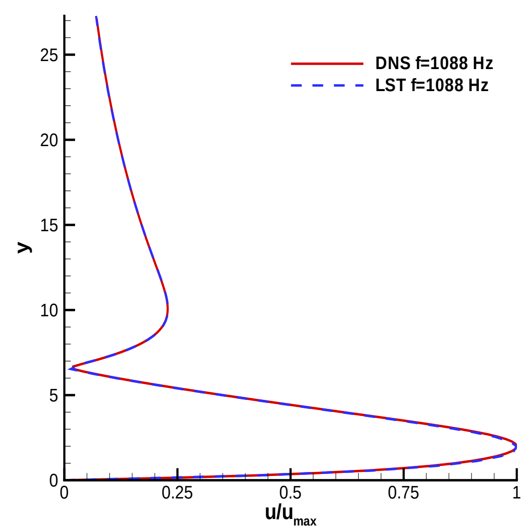


Figure 3: $u$ versus $y$ at $x = 3.27$, $F = 7.9$

The amplification rates as well as the shape of the $u(y)$-profile of the instability wave observed in the DNS agree very well with those predicted by linear stability theory (LST) which ensures that the observed wave really is a Tollmien-Schlichting wave. In the vicinity of the surface roughness a mixture of different disturbances is present, so that the initial amplitude of the generated TS-wave (which is necessary for the calculation of receptivity coefficients) can not be identified directly. Therefore a 'reference simulation' is needed where the amplitude development of a TS-wave generated upstream of the roughness by blowing and suction at the wall is simulated. The amplitude development of the two TS-waves downstream of the roughness is determined by the stability characteristics of the flow. So the amplitude of the TS-wave created by the interaction of the sound wave with the roughness in the vicinity of the roughness can be found by matching the amplitudes of the two simulations downstream of the roughness as illustrated in figure 4.

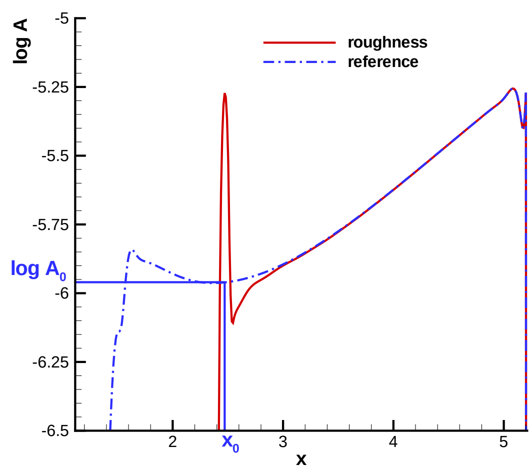


Figure 4: Calculation of the amplitude of the TS-wave at the location of the surface non-uniformity using two DNS.

# 4 VALIDATION

## 4.1 Comparison with a Theoretical Approach by Choudhari & Streett

To check the validity of the code for 2D receptivity simulations various test calculations varying the frequency of the sound wave and the location of the surface non-uniformity relative to the leading-edge were performed and the results were compared with results obtained by Choudhari & Streett (1994) [6] with 'Finite-Reynolds-Number-Theory' including a first-order model for the roughness.

As a measure for the receptivity of the boundary layer for a particular combination of wall inhomogeneity and free stream perturbation Choudhari & Streett introduced the so called 'efficiency function' $\Lambda_u$ which is defined as follows:

$$\Lambda_u = \frac{A_0}{A_{ac}\tilde{H}(\alpha_{TS})} \tag{1}$$

Here $A_0$ represents the amplitude of the TS-wave (maximum of $u$ versus $y$) at the location of the surface non-uniformity (see figure 4), $A_{ac}$ the amplitude of the sound wave and $\tilde{H}(\alpha_{TS})$ the Fourier transform of the shape of the roughness element, evaluated at the complex wave number $\alpha_{TS}$ of the instability wave.

$$\tilde{H}(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\alpha x} h(x)\, dx \qquad (2)$$

For small enough roughnesses the DNS results with the novel high-order wall model agree very well with the calculations of Choudhari & Streett as shown in figure 5.

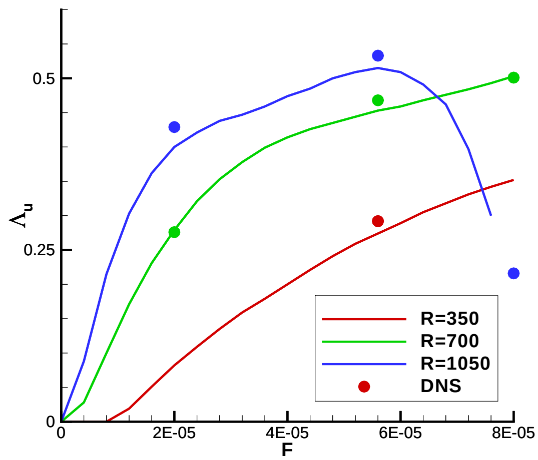


Figure 5: Magnitude of the efficiency function $\Lambda_u$ versus the nondimensional frequency parameter $F$ for different roughness locations $R = U_\infty \delta_1 / 1.72\nu$

## 4.2 Comparison with Wind-Tunnel Experiments

The experiments on the acoustic receptivity of a 2D laminar boundary layer on an airfoil in the presence of a 3D surface non-uniformity (see [13]) were carried out in the Laminar Wind Tunnel of our institute, which has a turbulence level of less than 0.02%.

The amplitude and phase part of the complex receptivity function $\tilde{G}_{av}$ (which means the ratio of the complex initial spectrum of the TS-waves $\tilde{A}_0$ to the acoustic amplitude $\tilde{A}_{ac}$ and the complex spectrum of the surface roughness $\tilde{A}_v$, evaluated at the frequency and streamwise wavenumbers of the created TS-waves, respectively) for 3D-waves with different propagation angles $\Theta$ ($\Theta = arctan(\beta/\alpha_r)$, $\beta$: spanwise wavenumber, $\alpha_r$: streamwise wavenumber) was calculated from the data measured in experiment as well as from the DNS data.

$$\tilde{G}_{av}(\alpha_r, \beta) = G_{av}(\alpha_r, \beta) \cdot e^{i\Phi_{av}(\alpha_r, \beta)} = \frac{\tilde{A}_0(\alpha_r, \beta)}{\tilde{A}_{ac} \cdot \tilde{A}_v(\alpha_r, \beta)} \tag{3}$$

with

$$\tilde{A}_0(\alpha_r, \beta) = \int A_0(z) \cdot e^{-i\beta z}\, dz \tag{4}$$

and

$$\tilde{A}_v(\alpha, \beta) = \int A_v(x, z) \cdot e^{-i(\alpha x + \beta z)}\, dxdz. \tag{5}$$

The agreement found between the DNS and the experimental results was very good, for different spanwise wavenumbers as well as for different frequencies.

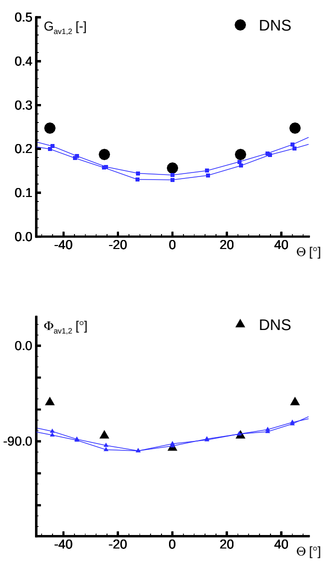


Figure 6: amplitude and phase part of the complex receptivity function for $f = 720Hz$

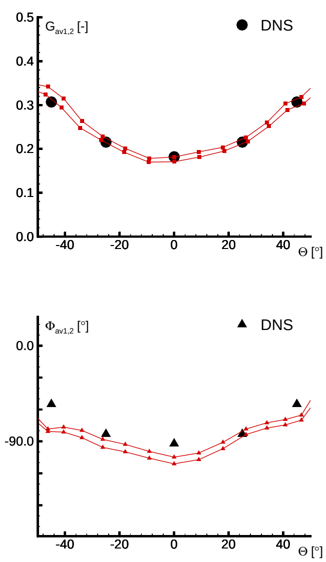


Figure 7: amplitude and phase part of the complex receptivity function for $f = 1088Hz$

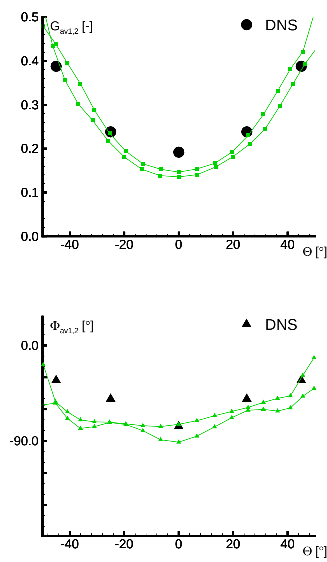


Figure 8: amplitude and phase part of the complex receptivity function for $f = 1562Hz$

This is shown in figures 6-8 where the amplitude and phase part of the complex receptivity function is plotted against the propagation angle for three different acoustic frequencies. The roughness acoustic receptivity was found to be lowest for 2D TS-waves and increases with propagation angle and frequency.

## 5 CONCLUSIONS

By means of direct numerical simulations (DNS) the acoustic receptivity of a Blasius boundary layer in the presence of a small 3D surface non-uniformity was studied. A new way of modelling the presence of a small roughness element on a flat plate without using a body-fitted coordinate system and without linearization was presented. By a comparison with results from a linear 2D receptivity theory for small enough surface non-uniformities, the correctness of this modelling was verified.

The 3D acoustic roughness receptivity calculations were compared with experiments carried out at the Laminar Wind Tunnel (LWT) of the IAG. The calculated receptivity coefficients showed a good quantitative agreement with the ones measured in the experiments. The scattering of the acoustic wave was found to be significantly more effective for 3D surface non-uniformities rather than for 2D ones. In the examined case the acoustic receptivity coefficients for all propagation angles increase with the acoustic frequency. This increase is stronger for 3D modes than for 2D ones. The phase delays of the generated TS-waves depend in contrast only weakly on the spanwise wavenumber and frequency.

## 6 ACKNOWLEDGEMENT

This work was performed under grant of the German Research Council (DFG) and is part of the research program "Transition". The contribution of Y. S. Kachanov was supported by SEW-Eurodrive and Volkswagen-Stiftung.

## REFERENCES

### References

[1] Goldstein, M.E.: "Scattering of acoustic waves into Tollmien-Schlichting waves by small streamwise variations in surface geometry", *J. Fluid Mech.*, Vol. **154**, pp. 509–529, (1985).

[2] Ruban, A.I.: "On the generation of Tollmien-Schlichting waves by sound", *Fluid Dyn.*, **19**, 709, (1985). (translation)

[3] Zhigulev, V. N. and Fedorov, A. V.: "Boundary-layer receptivity to acoustic disturbances", *J. Appl. Mech. Tech. Phys.*, **28**, 28, (1987).

[4] Saric, W.S., Hoos, J.A. and Radeztsky, R.H.: "Boundary-layer receptivity of sound with roughness", *ASME*, New York, FED.-Vol. **114**, pp. 17–22, (1991).

[5] Crouch, J.D.: "Localized receptivity of boundary layers", *Phys. Fluids*, A **4**, pp. 1408–1414, (1992).

[6] Choudhari, M. and Streett, C.L.: "A finite Reynolds-number approach for the prediction of boundary-layer receptivity in localized regions", *Phys. Fluids*, A **4**, pp. 2495–2514, (1992).

[7] Kobayashi, R., Fukunishi, Y., Ishii, T. and Reed, H.L.: "The receptivity of flat-plate boundary-layers to freestream sound", *Proc. 5th Int. Symp. Comp. Fluid Dyn. Sendai*, Vol. II , JSCFD, (1993).

[8] Zhou, M.D., Liu, D.P., Blackwelder, R.F.: "An experimental study of receptivity of acoustic waves in laminar boundary layers", *Exp. in Fluids*, **17**, pp. 1–9, (1994).

[9] Crouch, J.D. and Spalart, P.R.: "A study of non-parallel and nonlinear effects on the localized receptivity of boundary layers", *J. Fluid Mech.*, Vol. **290**, pp. 29–37, (1995).

[10] Rist, U. and Fasel, H.: "Direct numerical simulation of controlled transition in a flat-plate boundary layer", *J. Fluid Mech.*, Vol. **298**, pp. 211–248, (1995).

[11] Casalis, G., Gouttenoire, C. and Troff, B.: "DNS investigation of roughness-induced localized receptivity", *DNS and LES of complex flows*, University of Twente, pp. 134–139, (1997).

[12] Saric, W.S. and White, E.B.: "Influence of high-amplitude noise on boundary-layer transition to turbulence", *AIAA Paper 98-2645*, (1998).

[13] Würz, W., Herr, S., Wörner, A. , Rist, U., Wagner, S. and Kachanov, Y. S.: "Study of 3D wall roughness acoustic receptivity on an airfoil", *Proc. IUTAM-Symp. Laminar-Turbulent Transition*, Sedona, AZ, USA 13.-17. Sept. 1999, H. Fasel, W. Saric (eds.), Springer Verlag, Berlin, Heidelberg (1999).

[14] Cullen, L.M. and Horton, H.P.: "Acoustic receptivity in boundary layers with surface roughness" *Proc. IUTAM-Symp. Laminar-Turbulent Transition*, Sedona, AZ, USA 13.-17. Sept. 1999, H. Fasel, W. Saric (eds.), Springer Verlag, Berlin, Heidelberg (1999).