

Tunnel-Wall Corrections at the Laminar Wind Tunnel

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The Standard-Tunnel-Wall Corrections which are applied at wind tunnels were formulated by Glauert in 1938 [1]. They are based on simple Potential Theory and are widely accepted. Modern computer codes allow the calculation of much more specialized corrections for an individual airfoil. But it is usual to favour the Standard Corrections because of the conformity of the test results. Only in special cases these extended corrections could be used.

The effects of the tunnel walls upon the flow over a model are separated in:

- (1) Solid Blockage: An increase in free-stream velocity around the model caused by the constriction of the flow.
- (2) Streamline curvature: The normal curvature of the free air-flow about a model is straightened by the tunnel walls so it appears to have more camber than it actually has. Accordingly an airfoil in the tunnel has more lift and moment about its quarter chord at a given angle of attack.
- (3) Wake blockage: The blockage due to the wake of the model.
- (4) Buoyancy: Variation of the static pressure by thickening of the boundary layers along the tunnel walls.

The closed test section of the LWK is a rectangle with the dimension 2,73m x 0,73m. The two dimensional models have a span of 0,73m and are mounted in an unusual vertical position. SO the "side-walls" for the model lie in horizontal position.

The Standard-Tunnel-Wall Corrections as applied in the LWK are the following:

Correction for Solid Blockage :

Glauert /1/ considered an infinite vertical row of images of a symmetrical body which represent the effect of the tunnel walls as shown in figure 1. For the mathematical approach as given in /2/ the potential function for a symmetrical body is given by

$$\omega = F(z) = V \cdot z + \frac{A_1}{z} + \frac{A_2}{z^2} + \dots + \frac{A_n}{z^n} \quad (1)$$

V is the free stream velocity and the coefficients A_1, A_2, \dots are complex. If the tunnel height is large compared to the size of the body, powers of $1/z$ greater than 1 may be neglected so

$$\omega = F(z) = V \cdot z + \frac{A_1}{z}$$

This operation is equivalent to replacing a body by a circular cylinder of which the doublet strength is $2\pi A_1$.

The term $\frac{A_1}{z}$ represents the disturbance to the free-stream flow. With

$$a^2 = \frac{A_1}{V} \quad (2) \quad a = \text{Radius of the circular cylinder.}$$

$$\omega = F(z) = V \cdot \left(z + \frac{a^2}{z}\right) \quad (3)$$

The complex velocity is

$$\bar{w} = \frac{dF(z)}{dz} = V \cdot \left(1 + \frac{a^2}{z^2}\right) = u - i \cdot v$$

with

$$z = r(\cos \varphi + i \cdot \sin \varphi)$$

$$\frac{\bar{w}}{V} = 1 - \left(\frac{a}{r}\right)^2 \cos 2\varphi + i \cdot \left(\frac{a}{r}\right)^2 \sin 2\varphi \quad (3a)$$

for

$$\varphi = \frac{\pi}{2} \quad \text{there is} \quad \cos 2\varphi = -1$$

thus

$$\frac{u}{V} = 1 + \frac{a^2}{r^2} \quad \text{with} \quad \Delta V = u \quad \text{the increase in free stream velocity is}$$

$$\frac{\Delta V}{V} = + \frac{a^2}{r^2} \quad (4)$$

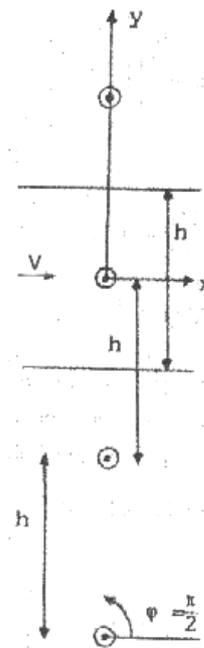


Figure 1

The velocity induced in the center of the cylinder by the reflected cylinders of one side is

$$\frac{\Delta V}{V} = \left(\frac{a}{r}\right)^2 + \left(\frac{a}{2r}\right)^2 + \dots + \left(\frac{a}{n \cdot r}\right)^2 = \left(\frac{a}{r}\right)^2 \left[1 + \frac{1}{4} + \frac{1}{9} + \dots\right] \quad (5)$$

for $n \rightarrow \infty$

$$\frac{\Delta V}{V} = \left(\frac{a}{r}\right)^2 \cdot \frac{\pi^2}{6}$$

The induced velocity from both sides of the cylinder is

$$\frac{\Delta V}{V} = \left(\frac{a}{r}\right)^2 \cdot \frac{\pi^2}{3} \quad (6)$$

Setting $r = h$ the height of the tunnel $h = 2.73$ m

with equation (2)

$$\Delta V = \frac{A_1}{h^2} \cdot \frac{\pi^2}{3} \quad (7)$$

Defining

$$\frac{\Delta V}{V} = \Lambda \cdot \sigma \quad (8)$$

with

$$\sigma = \frac{\pi^2}{48} \cdot \left(\frac{c}{h}\right)^2 \quad (9) \quad c = \text{chord of the model}$$

and

$$\Lambda = \frac{16 \cdot A_1}{c^2 \cdot V} \quad (10)$$

The factor σ depends only on the size of the model and is easily calculated.

A_1 is calculated from the potential function of a symmetrical airfoil equation (1). Where the

surface length is

$$ds = \sqrt{1 + \left(\frac{dy_t}{dx}\right)^2}$$

With the velocity distribution v/V over the airfoil with its camber removed with coordinates

x, y_t where $\frac{dy_t}{dx}$ is the slope of the surface at any point of the ordinate y_t

$$\Lambda = \int_0^1 \frac{16}{\pi} \cdot \frac{v}{V} \sqrt{1 + \left(\frac{dy_t}{dx}\right)^2} \cdot d\left(\frac{x}{c}\right) \quad (11)$$

For details to obtain this expression see reference /2/.

Values of Λ as function of airfoil thickness t/c are shown in fig. 2 for different NACA airfoil series.

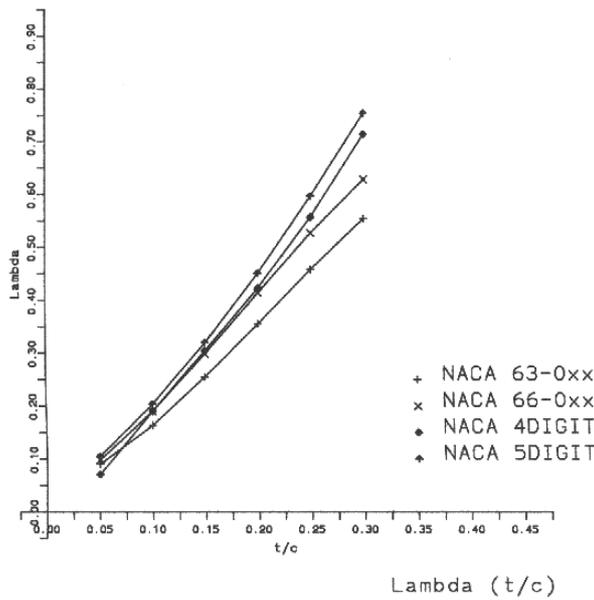


Figure 2

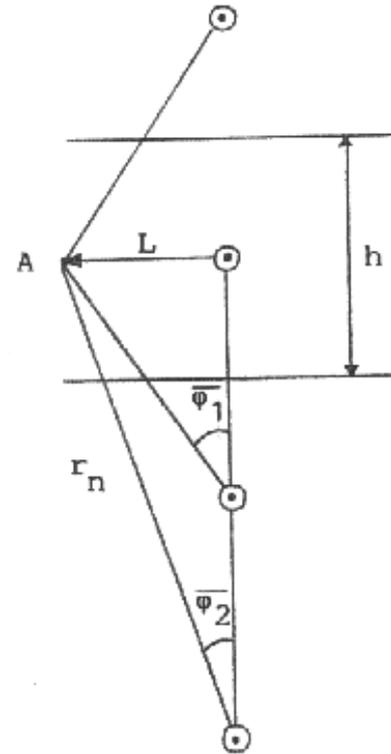


Figure 3

Correction of the measured tunnel velocity:

The tunnel velocity is indicated by a static pressure orifice located 2,0 meters upstream of the model on the center line of the tunnel. Caused by blocking an error exists in the measured tunnel velocity. The airfoil is simulated by a circular cylinder in the center of the turn table in the test section.

By the row of circular cylinders simulating the tunnel walls an additional velocity is induced at the position of the center see figure 3.

The complex velocity for a circular cylinder with radius a is (equation (3a))

$$\frac{\bar{w}}{V} = 1 - \left(\frac{a}{r}\right)^2 \cdot \cos 2\varphi + i \cdot \left(\frac{a}{r}\right)^2 \cdot \sin 2\varphi \quad (12)$$

$$\frac{\bar{w}}{V} = \frac{u}{V} + i \frac{v}{V}$$

For the central cylinder is $\varphi = \pi$. At the static orifice at position A in a distance L upstream of the center the velocity is

$$\frac{\Delta u_1}{V} = 1 - \left(\frac{a}{L}\right)^2 \quad (13)$$

The velocity induced by the row of cylinders is

$$\frac{\Delta u_2}{V} = 2 \cdot \left(\frac{a}{h}\right)^2 \cdot \left[\left(\frac{h}{r_1}\right)^2 \cdot \cos 2\bar{\varphi}_1 + \left(\frac{h}{r_2}\right)^2 \cdot \cos 2\bar{\varphi}_2 + \dots \right] \quad (14)$$

with h = tunnel height and r = distance of cylinders to A

and
$$\bar{\varphi} = \varphi - \frac{\pi}{2} \quad \bar{\varphi} = \arctg \frac{L}{n \cdot h}$$

and
$$\frac{h}{r_n} = \frac{h}{\sqrt{L^2 + (nh)^2}} \rightarrow \frac{1}{n^2} \text{ for large } n$$

$\bar{\varphi}$ becomes small with n so it is sufficient to use equation (14) only for $n \leq 5$
For the remaining rest equation (7) from the blocking correction minus these 5 terms can be used:

$$\frac{\Delta u_3}{V} = 2 \left(\frac{a}{h}\right)^2 \cdot \left[\frac{\pi^2}{6} - \sum_1^n \frac{1}{n^2} \right] \quad n \leq 5 \quad (15)$$

The total velocity induced velocity at Point A is
$$\frac{\Delta u}{V} = \frac{\Delta u_1}{V} + \frac{\Delta u_2}{V} + \frac{\Delta u_3}{V} \quad (16)$$

$$\frac{\Delta u}{V} = -\left(\frac{a}{L}\right)^2 + 2\left(\frac{a}{h}\right)^2 \cdot \sum_1^5 \frac{h}{r_n} \cdot \cos 2\bar{\varphi}_n + 2\left(\frac{a}{h}\right)^2 \cdot \left[\frac{\pi^2}{6} - \sum_1^5 \frac{1}{n^2} \right] \text{ with } a^2 = \frac{A_1}{V} \text{ eq. (2)}$$

$$\frac{\Delta u}{V} = \frac{A_1}{V} \left[-\frac{1}{L^2} + 2\left(\frac{1}{h}\right)^2 \cdot \sum_1^5 \left(\frac{h}{r_n}\right)^2 \cdot \cos 2\bar{\varphi}_n + 2\left(\frac{1}{h}\right)^2 \cdot \left[\frac{\pi^2}{6} - \sum_1^5 \frac{1}{n^2} \right] \right] \quad (17)$$

From the calculation of the form factor Λ

$$\Lambda = \frac{16 \circ A_1}{c^2 \cdot V} \quad (10) \quad \text{or} \quad \frac{A_1}{V} = \frac{\Lambda \cdot c^2}{16}$$

$$\frac{\Delta u}{V} = \frac{\Lambda \cdot c^2}{16} \cdot \left[-\frac{1}{L^2} + 2\left(\frac{1}{h}\right)^2 \cdot \sum_1^5 \left(\frac{h}{r_n}\right)^2 \cdot \cos 2\bar{\varphi}_n + 2\left(\frac{1}{h}\right)^2 \cdot \left[\frac{\pi^2}{6} - \sum_1^5 \frac{1}{n^2} \right] \right] \quad (18)$$

Setting
$$\xi = \frac{c^2}{16} \cdot \left[-\frac{1}{L^2} + 2\left(\frac{1}{h}\right)^2 \cdot \sum_1^5 \left(\frac{h}{r_n}\right)^2 \cdot \cos 2\bar{\varphi}_n + 2\left(\frac{1}{h}\right)^2 \cdot \left[\frac{\pi^2}{6} - \sum_1^5 \frac{1}{n^2} \right] \right] \quad (19)$$

Calculated with the tunnel dimensions:
$$\xi = -0.00335 \circ c^2 \quad (20)$$

Values for ξ (c) are tabulated in table 1.

The induced velocity can be calculated by

$$\frac{\Delta V}{V} = \Lambda \cdot \xi \quad (21)$$

The tunnel velocity corrected for static pressure error in the tunnel wall orifice is

$$V' = V'' \cdot (1 + \Lambda \cdot \xi) \quad (22)$$

V'' being the velocity as indicated by the static orifice.

$c [m]$	ξ
0,5	-0,00085
0,7	-0,00167
1,0	-0,00340

Table 1

Correction for streamline curvature:

The tunnel walls prevent the normal curvature of the free air that occurs about a lifting body and it appears to have more camber than it actually has. Accordingly the model in the wind tunnel has more lift and moment about the quarter chord at a given angle of attack.

According to thin airfoil theory the airfoil is approximated by a single vortex at its quarter-chord point in the tunnel center ($x=0, y=0$). The wind tunnel walls are simulated by an infinite vertical row of vortices with distance h and of alternate sign above and below the real vortex (see figure 4). The horizontal velocities induced at a position x on the tunnel center line cancel but the vertical components add. At the position of the vortex the induced vertical component is zero and changes sign. In a closed tunnel the curvature of the flow must be so that there is no flow through the tunnel walls.

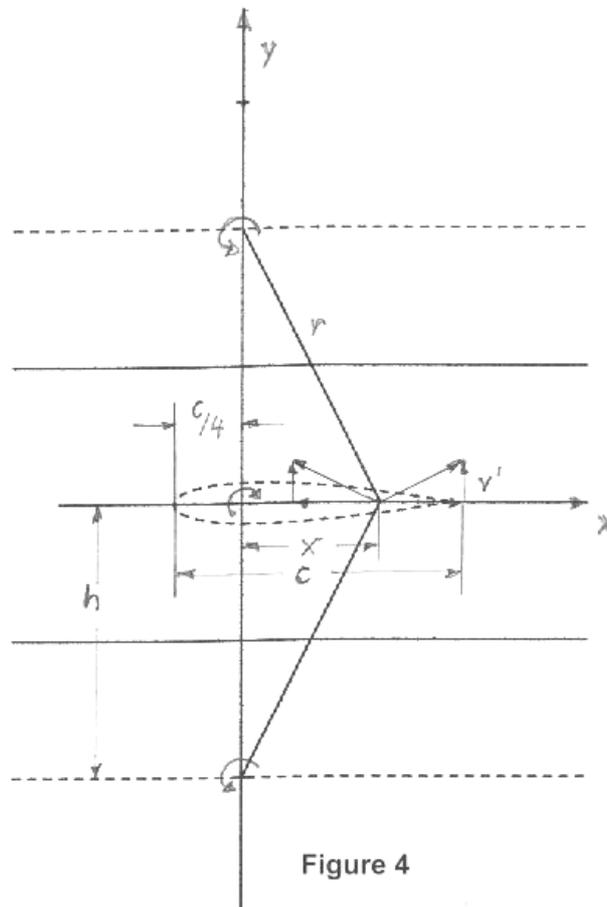


Figure 4

Consider the first pair of vortices. From vortex theory follows:

$$w = \frac{\Gamma}{2\pi \cdot r} = \frac{\Gamma}{2\pi} \cdot \frac{1}{\sqrt{h^2 + x^2}} \quad \frac{v'}{w} = \frac{x}{r} = \frac{x}{\sqrt{h^2 + x^2}} \quad v' = \frac{\Gamma}{2\pi} \frac{x}{\sqrt{h^2 + x^2}} \cdot \frac{1}{\sqrt{h^2 + x^2}}$$

$$v' = \frac{\Gamma}{2\pi} \cdot \frac{x}{h^2 + x^2} \quad (23)$$

v' is the vertical velocity at position x on the center line. Assuming reasonable values for x and h shows that the upwash angle varies almost linearly along the chord, hence the stream curvature is nearly circular.

A rough analysis is given in /5/. It bases on the fact that the upwash correction v'/V at its mid-chord is the correction $\Delta\alpha$ for the angle of attack.

At the mid-chord at $x=c/4$ the upwash induced by the first two images nearest to the real airfoil is

$$v' = 2 \cdot \frac{\Gamma}{2\pi} \cdot \frac{c/4}{h^2 + (c/4)^2} \quad (24) \quad \text{the angular correction is } \frac{v'}{V}$$

and since $\Gamma = c_l \cdot \frac{V}{2} \cdot c$

$$\Delta\alpha = \frac{v'}{V} = \frac{1}{8\pi} \cdot \frac{c^2}{h^2 + (c/4)^2} \cdot c_l'$$

$(c/4)^2$ can be neglected compared to h^2

$$\Delta\alpha = \frac{v'}{V} = \frac{1}{8\pi} \cdot \frac{c^2}{h^2} \cdot c_l' \quad (25) \quad \text{using again } \sigma = \frac{\pi^2}{48} \circ \left(\frac{c}{h}\right)^2 \quad (9)$$

we get

$$\Delta\alpha = \frac{6\sigma}{\pi^3} \cdot c_l' \quad (26)$$

The second pair of vortices being twice as far from the airfoil will be roughly one-fourth as effective, and the third pair one-ninth. So for the images above and below we have

$$\Delta\alpha = \frac{6\sigma}{\pi^3} \cdot \left(1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} \dots\right) c_l' \quad (27)$$

$$\Delta\alpha = \frac{6\sigma}{\pi^3} \cdot \frac{\pi^2}{12} \cdot c_l' = \frac{1}{2\pi} \cdot \sigma \cdot c_l' \quad (28)$$

and the additive lift correction is

$$\Delta c_l = 2\pi \cdot \Delta\alpha = \sigma \cdot c_l' \quad (29)$$

the additive momentum correction is

$$\Delta c_{m_{\frac{c}{4}}} = -\frac{\sigma}{4} \cdot c_l' \quad (30)$$

A more thorough analysis is given by Allen and Vincenti /4/ they spread the vorticity out along the airfoil chord instead of concentrating it at the quarter chord. Lift and moment values of the simple analysis remain the same but the angle-of-attack correction becomes

$$\Delta\alpha = \frac{1}{2\pi} \cdot \sigma \cdot \left(c_l' + 4c_{m_{\frac{c}{4}}}'\right) \quad (31)$$

As already mentioned the value of the angular correction v'/V at mid-chord is precisely the correction $\Delta\alpha$ for the angle of attack.

Lift interference on two-dimensional wings is also treated by H. C. Garner, E. W. E. Rogers, W. E. A. Acum and E. C. Maskell / /.

3) Wake blockage:

The blocking due to the wake of the model is neglected as this effect is estimated to be small. The drag in the LWK is derived from the loss of total pressure in the wake.

If the drag is measured by a balance the wake gradient effect on C_d according to /4/ is

$$\Delta c_{d,wb} = \Lambda \cdot \sigma \quad (32)$$

(4) Buoyancy:

In Wind tunnels with closed test sections a thickening of the boundary layers along the tunnel walls causes a diminution of the jet area which results in a variation of static pressure along the tunnel axis. This effect is nearly eliminated by a divergence of the tunnel walls opposite to the suction and pressure sides of the model /7/.

The standard wind tunnel corrections.

Finally the standard wind tunnel corrections applied at the LWK are /2/4/5:

$$c_l = (1 - 2\Lambda \cdot (\sigma + \xi) - \sigma) \cdot c_l' \quad (33)$$

$$c_{\frac{m_t}{4}} = (1 - 2\Lambda \cdot (\sigma + \xi)) \cdot c_{\frac{m_t}{4}}' + \left[\sigma \cdot \left(\frac{c_l'}{4} \right) \right] \quad (34)$$

$$\alpha_0 = (1 + \sigma) \cdot \alpha_0' + \frac{4\sigma \cdot c_{\frac{m_t}{4}}'}{dc_l' / d\alpha_0'} - \sigma \cdot \alpha_{c_{10}}' \quad (35)$$

The primed quantities refer to the coefficients measured in the tunnel.

In the last two equations the last terms can be neglected for chords less than 1 meter.

The wall effect on the drag is merely that corresponding to an increase of the tunnel speed:

$$c_d = (1 - 2\Lambda \cdot (\sigma + \xi)) \cdot c_d' \quad (36)$$

The corrected free stream velocity is

$$V = V' \cdot (1 + \Lambda \cdot (\sigma + \xi)) \quad (37) \quad \text{and the dynamic pressure} \quad q = q' \cdot (1 + 2 \cdot \Lambda \cdot (\sigma + \xi)) \quad (38)$$

Similar corrections are applied to the pressure distribution around the airfoil.

Further information on wind tunnel corrections may be found in ref. /5/ and /6/.

Standard Wind Tunnel Corrections for the LWK.

$$c_l = K_{C_l} \cdot c_l' \quad K_{C_l} = (1 - 2\Lambda \cdot (\sigma + \xi) - \sigma) \quad (39)$$

$$c_d = K_{C_d} \cdot c_d' \quad K_{C_d} = (1 - 2\Lambda \cdot (\sigma + \xi)) \quad (40)$$

$$c_{\frac{m}{4}} = K_{C_m} \cdot c_{\frac{m}{4}}' \quad K_{C_m} = (1 - 2\Lambda \cdot (\sigma + \xi)) \quad (41)$$

$$\alpha = K_{\alpha} \cdot \alpha' \quad K_{\alpha} = 1 + \sigma \quad (42)$$

The corrections for different chord lengths c of the wind tunnel models are summarized in tables:

	c (m)	σ	Λ	$\Lambda\sigma$	K_{C_l}	K_{C_d}
h=2.73m	1,0	0.026	0.3	0.0079	0.958	0.984
	0,85	0.019	0.3	0.0059	0.970	0.989
	0,6	0.009	0.3	0.0028	0.985	0.994

The influence of the form-factor Λ .

For an airfoil model with a thickness to chord ratio of 0.1 the form-factor Λ is ~ 0.18 , for a thickness of $t/c = 0.25$ $\Lambda \sim 0.5$.

Chord length $c=1,0$ m:

Λ	K_{C_l}	$K_{C_d} = K_{C_m}$
0.2	0.963	0.990
0.3	0.958	0.984
0.5	0.948	0.974

Chord length $c=0,6$ m:

Λ	K_{C_l}	$K_{C_d} = K_{C_m}$
0.2	0.987	0.996
0.3	0.985	0.994
0.5	0.981	0.991

These wind tunnel corrections are applied during calibration of the test equipment. By this way the corrected coefficients are plotted and stored during the tests.

Figures 5 and 6 show the influence of the Standard Wind Tunnel corrections on the polar of an airfoil with 16% thickness and chords of 0.6 and 1.0 m respectively.

The factors K_{C_l} , K_{C_d} , K_{C_m} and K_{α} are constant for all α and Reynolds numbers.

For airfoils with several components as flaps and slats the form factor Λ must be estimated. Experience and comparison with extended wind tunnel corrections proved however that these Standard Wind Tunnel corrections are tolerable in most cases. They are used in similar form in nearly all wind tunnels.

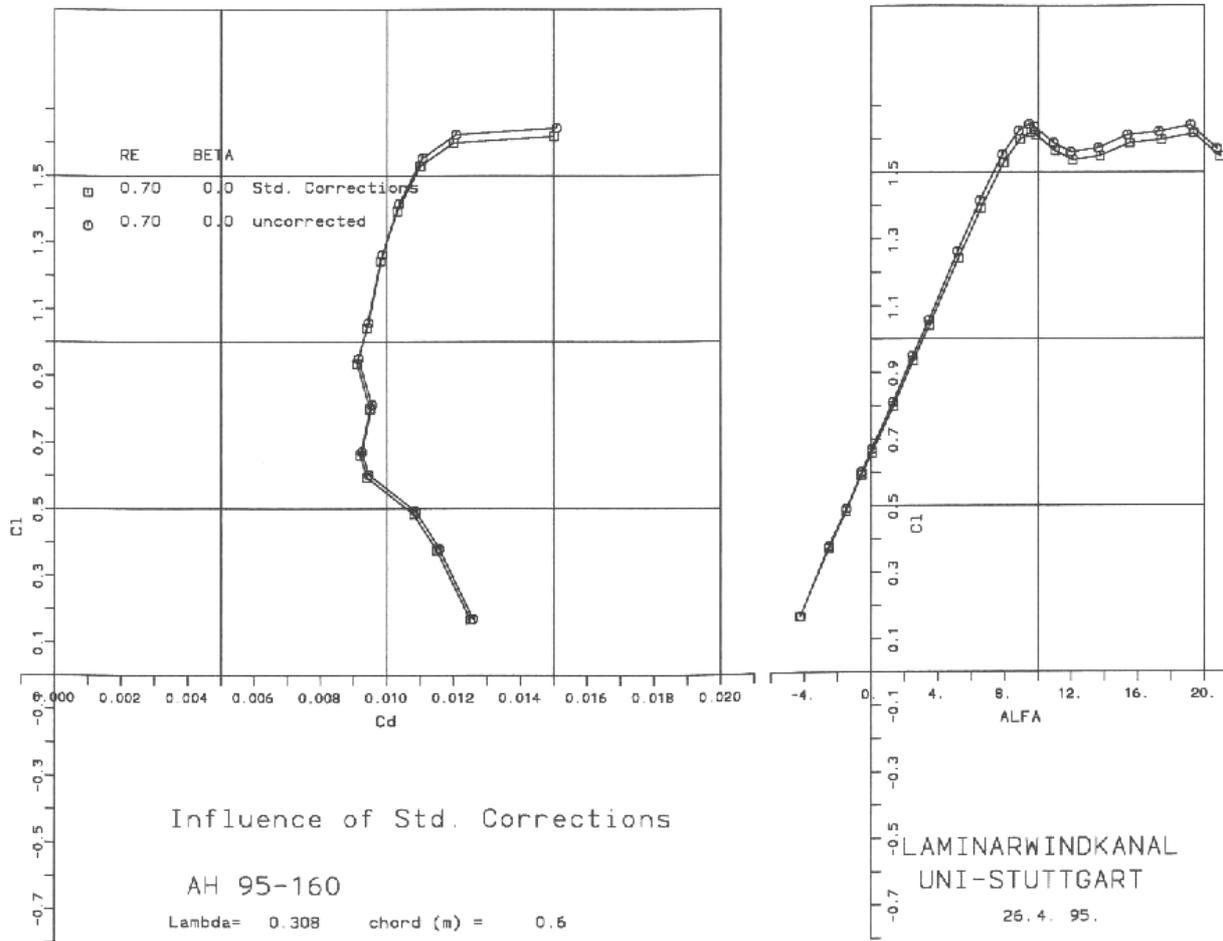


Figure 5

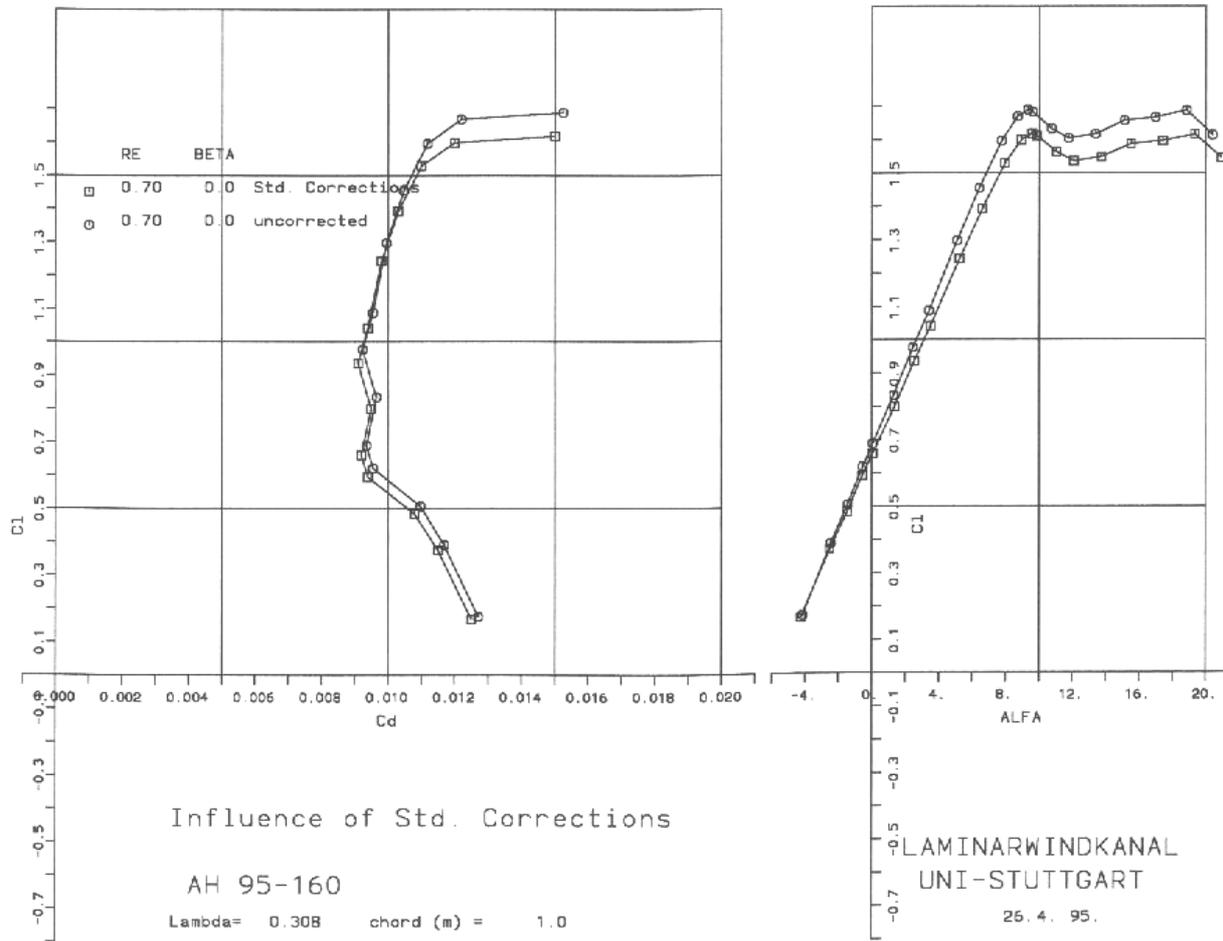


Figure 6

The effectiveness of the standard wind tunnel corrections.

Modern computer codes allow to calculate the flow about airfoil systems with two or more components. With the MSES-code by M. Drela /8/ airfoil systems with consideration of the boundary layers and the wake can be calculated. The wind tunnel walls can be simulated by a fixed border of the grid. Boundary layers are considered by addition of their displacement thickness to the contours of the model components.

Polars for a prescribed set of angle of attack or lift coefficients can be calculated for prescribed Reynolds numbers.

Potential flow conditions are calculated for Reynolds number $Re = 0$. By this way the influence of the boundary layers and wakes can be demonstrated.

Calculations on some 25 airfoils with different thickness, design lift coefficients as well as airfoils with deflected flaps and up to three components were performed in free air and between solid tunnel walls. Reynolds numbers varied from zero to 4 millions /9/.

As both cases are calculated by the same program at least relative consistency can be awaited.

Corrections the factors $F_{C_i} = c_i(\text{free})/c_i(\text{tunnel})$ can be calculated at equal incidence for the coefficients c_l, c_d, c_m . In contrast to the standard corrections these factors can vary with the angle of attack.

Figure 7 shows polars for an airfoil with a thickness to chord ratio of 15.5% and a chord length c of 0.6 meters for 3 Million Reynolds number. If one assumes the polar calculated between tunnel walls to be a "measured" polar in a wind tunnel this polar corrected with the standard corrections should agree with the polar calculated in free air. The differences between the curves "tunnel" and "tunnel with standard corrections" are acceptable for C_l and C_d larger differences show in c_m . Even the polars in figure 8 for the large chord of $c = 1,0$ m show good agreement. Figures 9 and 10 show polars for an airfoil with 30% thickness for 1 million Reynolds number. Differences are larger for the $c = 1,0$ meters.

Chord lengths between 0.5 and 0.85 meters ($h/c > 3,3$) are usual for the models in the LWK. The standard wind tunnel corrections showed to be acceptable in most cases.

In doubt a MSES calculation could be performed to obtain extended wind tunnel corrections.

Modern literature on wind tunnel corrections: /11/ 12/

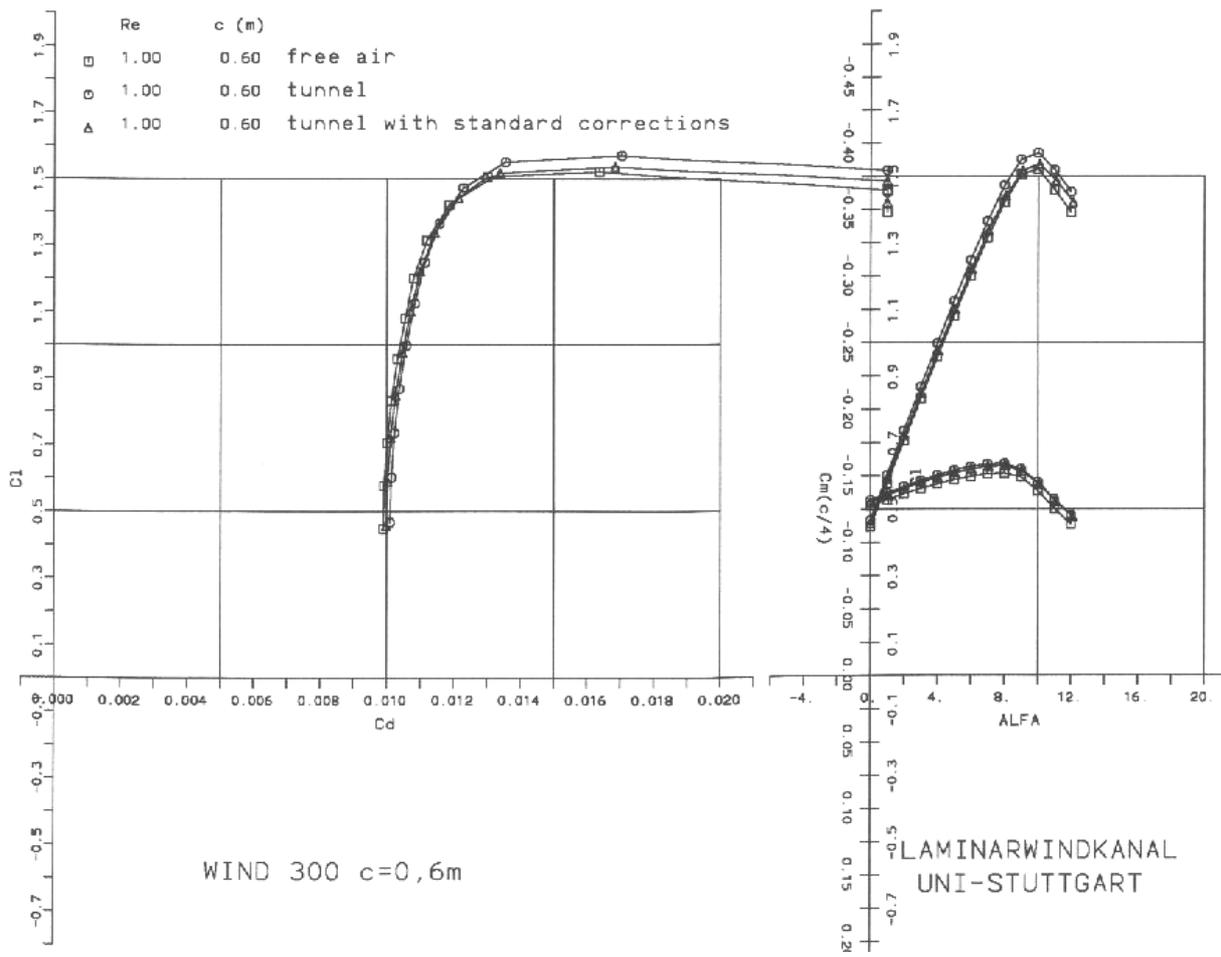


Figure 7

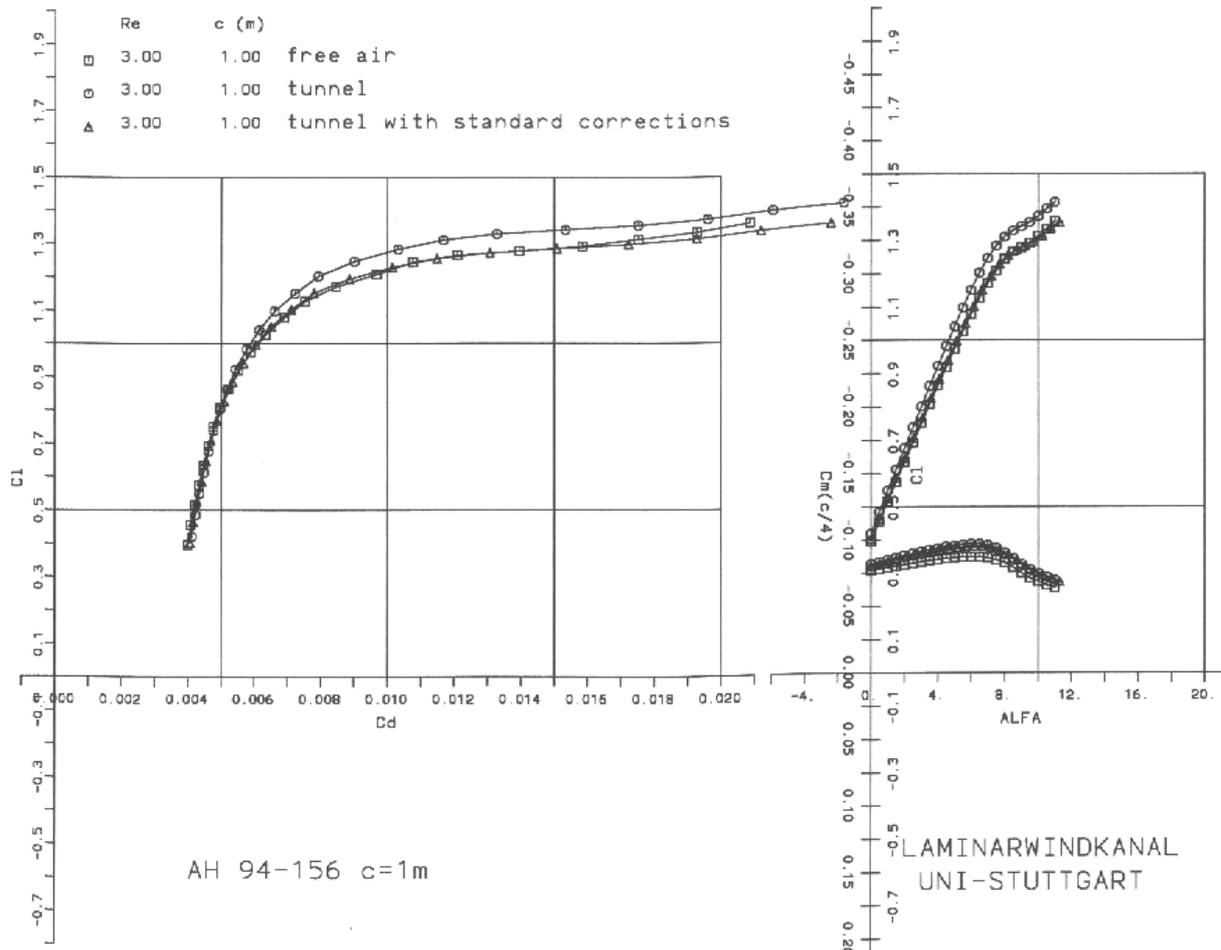


Figure 8

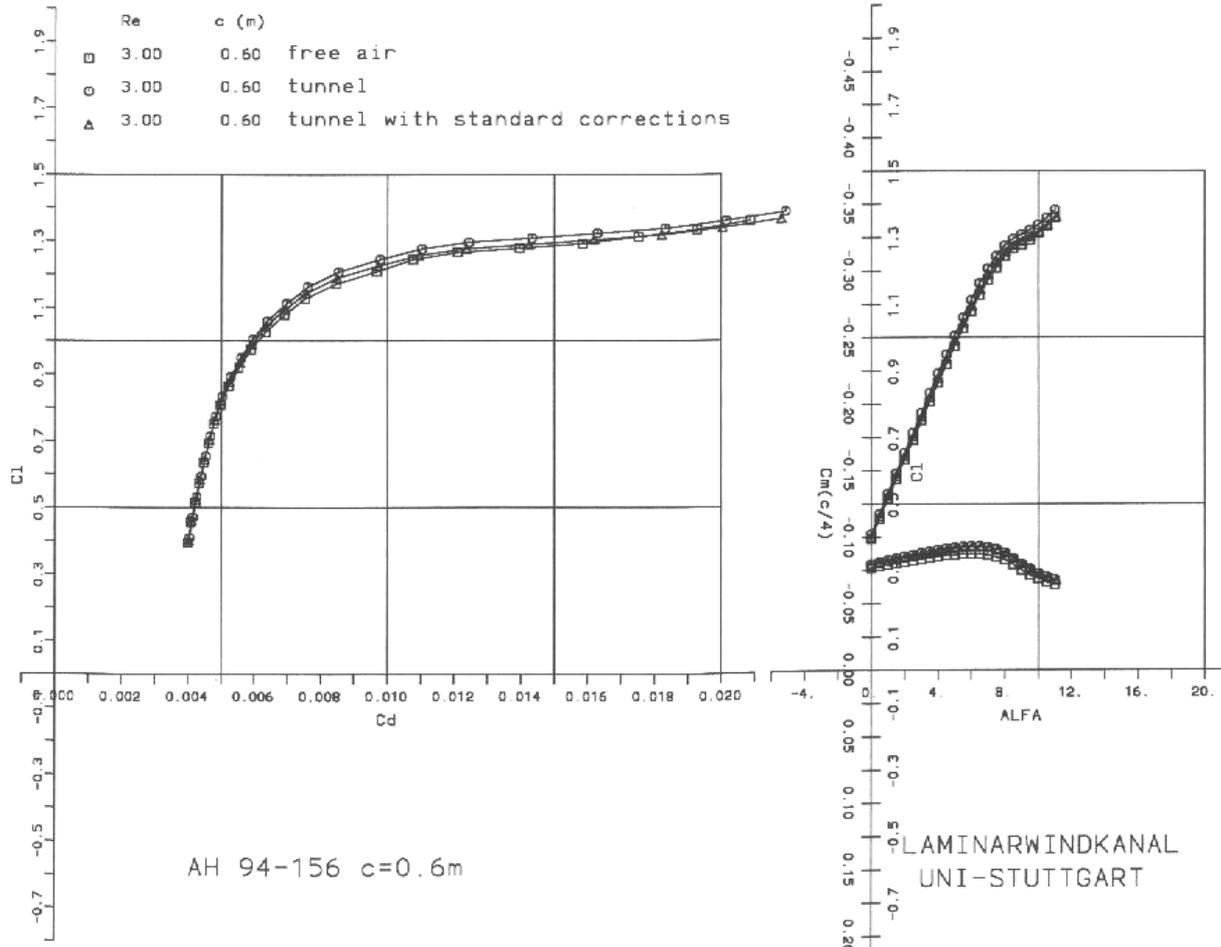


Figure 9

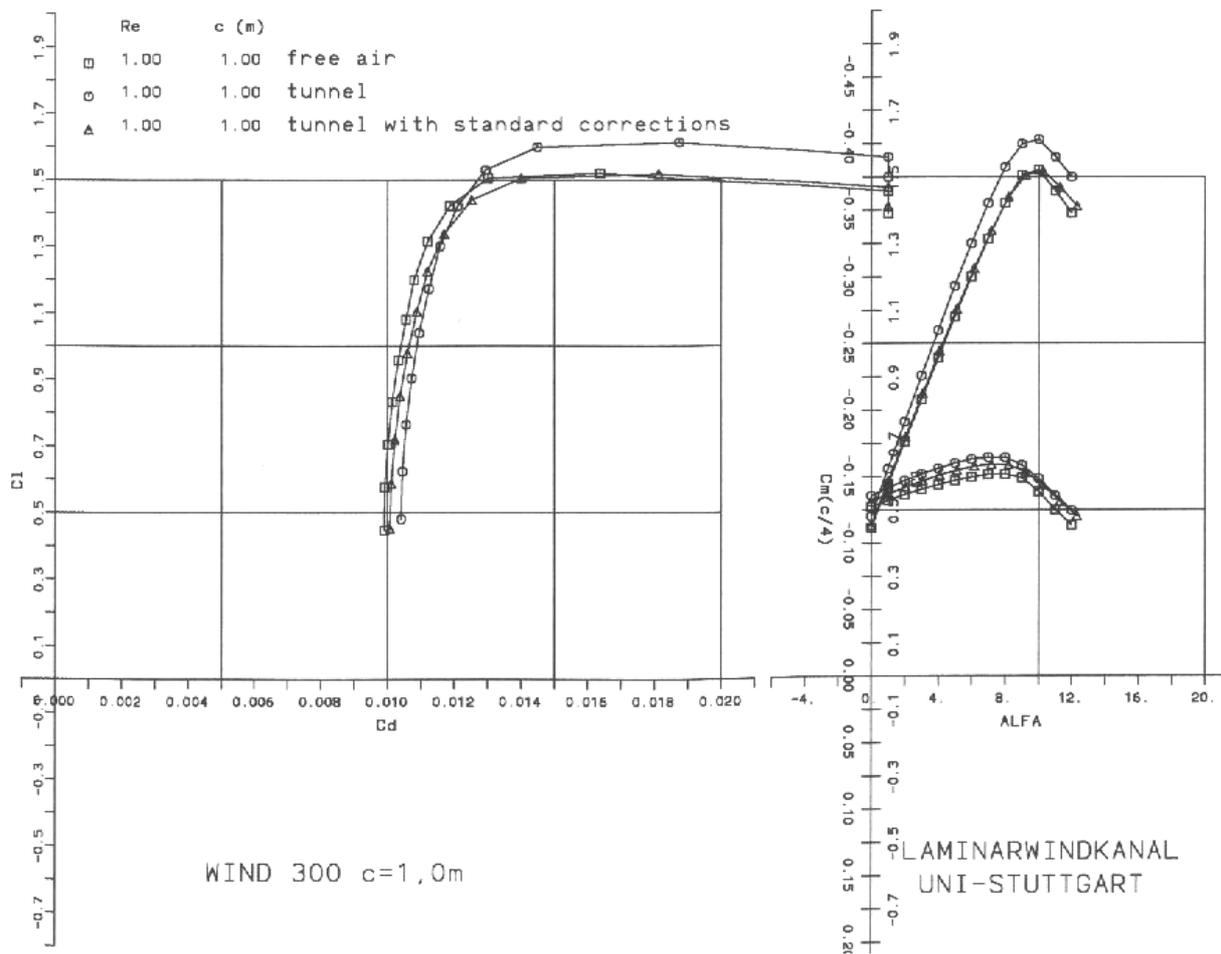


Figure 10

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