Sensitivity to base-flow variation of a streamwise corner flow

Alizard Frédéric *, Robinet Jean-Christophe * and Rist Ulrich +.

Abstract The stability of flow formed by intersection of two perpendicular flat-plates is revisited through a study of the sensitivity to the base flow variation. After a brief presentation of the asymptotic regime, sensitivity functions underlying corner mode (concentrated close to the intersection) and Tollmien-Schlichting modes, with different obliqueness angles, are computed. With this consideration, associated mechanisms as well as active regions are identified, which further confirm that the sensitivity area of the corner mode arises along the intersection of flat plates. Then, an optimization technique shows that a small deviation of the reference field in the area of uncertainty observed in experiments leads to decrease critical Reynolds number. A hypothesis based on the onset of an inflectional mechanism is thus proposed to explain the experimental results.

1 Introduction

Viscous flow along a corner formed by intersection of two semi-infinite perpendicular flat plates has been under investigation for several decades. In particular, experimental results highlighted a transitional Reynolds number based on the distance from the leading edge of about 10^4 which is much lower than the critical Reynolds number of the classical Blasius flat plate boundary layer $\approx 10^5$ [2]. Furthermore, local linear stability studies didn't allow to explain the experimental results [1]. Nevertheless, although the theory provides that the instability of a zero pressure gradient corner layer is dominated by the classical Tollmien-Schlichting (noted TS hereafter) viscous modes, an inviscid mode strongly localized in the corner is also observed. Moreover, experiments exhibit a large variety of flows really close to the corner and some discrepancies between the theoretical base flow and the experi-

* SINUMEF laboratory 151 Boulevard de l'Hopital 75013 Paris, e-mail: frederic.alizard@paris.ensam.fr and + Pfaffenwaldring 21, 70569 Stuttgart e-mail: rist@iag-uni.stuttgart.de

Fig. 1 Numerical solution of the self-similar corner equations. The shaded area indicates the region where a hypothetical laboratory uncertainty might appear.

mental data. Consequently, the purpose of this paper is to reconsider the problem through a stability analysis taking into account a certain degree of uncertainty associated with the base-flow in Figure 1. After a brief presentation of the theory, it will be shown that a small deviation of the base flow close to the corner allows to destabilize the inviscid mechanism derived from the corner mode.

2 Temporal asymptotic linear stability.

Reference lengths are based on the maximum streamwise velocity and the distance from the leading edge. Distubances $\mathbf{q}_p =^t (u_p, v_p, w_p, p_p)$ are assumed of the following form:

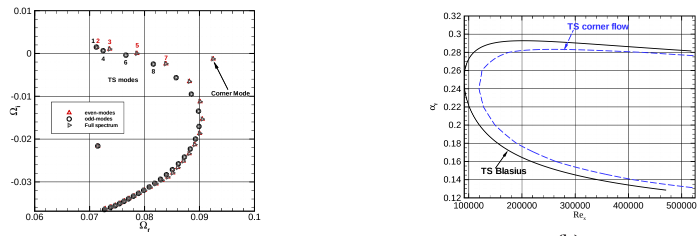
$$\mathbf{q}_p = \hat{\mathbf{q}}\,(y,z)\,e^{[i(\alpha x - \Omega t)]} \tag{1}$$

The space and time behaviour of a small perturbation is thus governed by the operator $\mathscr{L}\left(\{\boldsymbol{\Omega}, \hat{\mathbf{q}}\}, \mathbf{U}\right) = 0$ derived from the linearized Navier-Stokes equations. In a temporal framework, the wave number is fixed real, thus the mode is allowed to grow temporally with the growth rate and the circular frequency equal to Ω_i and Ω_r, respectively. The system of equations reduces to a large generalized eigenvalue problem which can be written :

$$(\mathscr{A} - i\Omega\mathscr{B})\,\hat{\mathbf{q}} = 0 \tag{2}$$

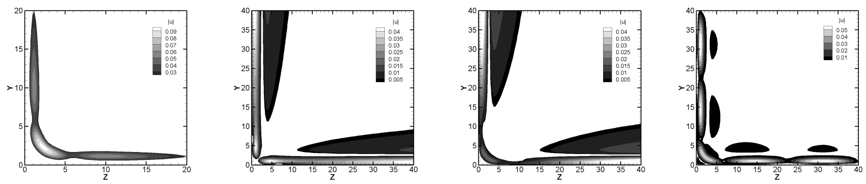
with $i\Omega$ eigenvalues and $\hat{\mathbf{q}}$ eigenfunctions.

The system (2) is discretized using a Chebyshev/Chebyshev spectral collocation method in the z and y directions. Classical no-slip boundary conditions are imposed at the walls and Neumann conditions in the far-field for the velocity components. The use of symmetry conditions leads to two possible solutions: even-symmetric and odd-symmetric modes. Finally, an Arnoldi algorithm, based on ARPACK, combined with a shift-invert method is used to approximate the most relevant part of the spectrum. A spectral grid (65×65) is employed in our next computations.



(a) (b)

Fig. 2 2(a) Temporal spectrum at $Re_x = 2.5 \times 10^5$ and $\alpha = 0.2$. 2(b) Neutral curve of the most unstable TS mode.



(a) $\hat{u}$: corner mode. (b) $\hat{u}$ of TS 1: A-S. (c) $\hat{u}$ of TS 2: S. (d) $\hat{u}$ of TS 4: S.

Fig. 3 TS and corner mode. S:symmetric, A-S: anti-symmetric 2.

Typical eigenvalue spectra for $Re_x = 2.5 \times 10^5$ and $\alpha = 0.2$ is depicted in Figure 2. A branch of eigenvalues may be observed which can be attributed to Tollmien-Schlichting (TS-) modes with different transverse wave lengths, i.e., different obliqueness angles with respect to the free-stream flow. The most unstable one corresponds to the classical (two-dimensional) TS-instability mode of the flat-plate boundary layer. Aside of this branch we get an isolated mode whose eigenfunction is dominantly concentrated near the corner line and rapidly decays along y and z. This is the so-called corner mode.

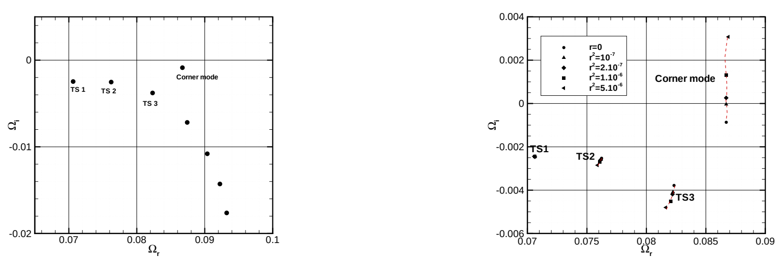
The most unstable TS-mode is compared with the classical TS mode of a Blasius boundary layer through a neutral curve in the plane (Re_x, α) in Figure 2(b). The influence of the corner on this specific mode is weak and it is stabilizing the flow. The critical Reynolds number based on the streamwise position Re_x equals to 1.17×10^5 which is consistent with the value for Blasius flow $\approx 9.1 \times 10^4$. Furthermore, the corner mode is observed to be always temporally stable in the parameters space which is analysed. These results are in good agreement with those of Parker & Balachandar [1] which validate our numerical methods.

From the above discussion, it seems clear that local theory cannot explain why experimentalists observe a premature laminar-turbulent transition of corner flows compared to flat-plate boundary layers. Therefore, in order to take into account the extreme sensitivity in the corner region observed in experimental measurements, we reconsider this problem through a stability analysis where a certain degree of uncertainty associated with the base flow is theoretically investigated.

3 Sensitivity analysis

3.1 Sensitivity functions

A sensitivity function $\mathbf{G_u}$ may be constructed by a projection of the perturbated operator $\mathscr{L}$ along the adjoint mode as follows: $\delta\Omega = \int_0^{L_y}\int_0^{L_z} {}^t\mathbf{G_u}\delta\mathbf{U}\ \mathrm{d}z\mathrm{d}y$ where $\delta\mathbf{U}$ is a small variation of $\mathbf{U}$ [3]. We take as a representative case the corner flow at $Re_x = 8\times 10^4$ and $\alpha = 0.18$. $\mathbf{G_u}$ is displayed in Figures 5 with respect to the modes depicted in the spectrum 4(a). One may observe that the corner mode is the



(a) Spectrum. (b) Optimized spectrum.

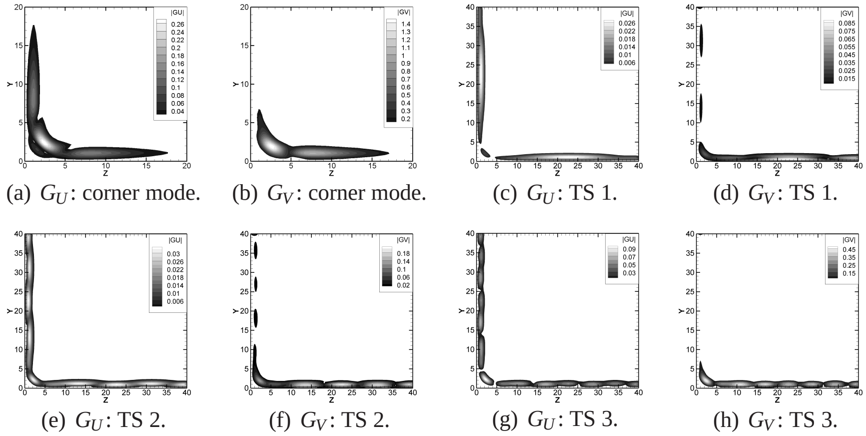
Fig. 4 $Re_x = 8\times 10^4$ and $\alpha = 0.18$.

most sensitive one to any base-flow modification around the uncertain area, which demonstrates that this last one is the best candidate to provide an explanation to the low-Reynolds number observed in experiments through a small base-flow deviation.

Therefore, on the basis of above results, it seems justified to further explore the influence of mean-flow modification having the most effect on the corner mode with respect to a deviation of a given magnitude.

3.2 Optimal deviation and physical mechanism

The small deviation of the mean-flow is measured through an energy-like norm:



(a) G_U: corner mode. (b) G_V: corner mode. (c) G_U: TS 1. (d) G_V: TS 1.

(e) G_U: TS 2. (f) G_V: TS 2. (g) G_U: TS 3. (h) G_V: TS 3.

Fig. 5 Sensitivity functions of the corner and TS modes. $Re_x = 8\times 10^4$ and $\alpha = 0.18$.

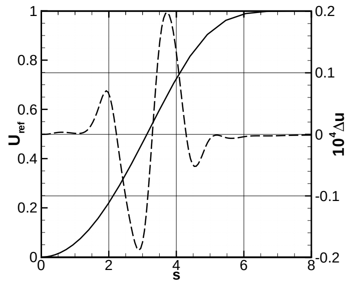
$$r^2 = \int_0^{L_y}\int_0^{L_z} \left(U - U_{ref}\right)^2 + \left(V - V_{ref}\right)^2 + \left(W - W_{ref}\right)^2 \text{ dy dz} \tag{3}$$

where $_{ref}$ refers to the theoretical base flow. We will focus on the deviation which maximizes the growth rate of this corner mode. A similar variational approach as by Bottaro *et al.* [3] is employed by introducing the functional

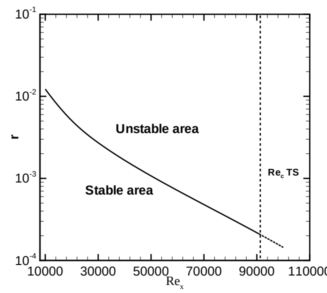
$$\begin{aligned}&\mathscr{H}(\mathbf{U},\lambda) = \Omega_i(\mathbf{U}) - \\ &\lambda\left(r^2 - \int_0^{L_y}\int_0^{L_z} \left(U - U_{ref}\right)^2 + \left(V - V_{ref}\right)^2 + \left(W - W_{ref}\right)^2 \text{ dy dz}\right)\end{aligned} \tag{4}$$

with λ a Lagrange multiplier. A constraint-optimization process classically employed in control theory is then introduced to maximize Ω_i by successive iterations on the control (U,V,W). An example of optimization is depicted in Figure 4(b). It appears that a small deviation of the base flow leads to destabilize the corner mode. Furthermore, from the velocity bissector profile displayed in Figure 6(a), it seems clear that the emerging instability mechanism derived from the optimal distorted base flow is strongly connected to the inflection point along the bissector.

Finally, we track the optimal deviation of the corner mode in the parameter space (α, r, Re_x). Figure 6(b) shows the neutral curve which indicates the lowest Reynolds number for which a positive amplification rate occurs over a reasonably small deviation of the ideal mean flow. Typically, here the modification is between 0.1% and 1%. It may be observed that the critical Reynolds number varies in inverse proportion to the disturbance amplitude. Nevertheless, it appears that even for a lower Reynolds number than $\approx 10^4$, i.e. one order of magnitude lower than the critical Reynolds number associated with the TS waves in supercritical regime, the optimal distorted base flow is able to experience exponential growth through an inviscid mechanism.



(a) Velocity profile along the bissector.



(b) Neutral curve with respect to (r, Re_x)

Fig. 6 Optimization results. In 6(a), the related optimal deviation for $r^2 = 2 \times 10^{-7}$ with respect to Figure 4(b) is ploted.

4 Conclusion

A study of sensitivity functions for TS and corner modes underlying a three-dimensional corner flow reveals the high sensitivity of the mean-flow close to the intersection. An optimization technique applied to the corner mode shows the influence of a weak deviation of the reference base-flow in the area of uncertainty of the latter. It illustrates the possibility of destabilizing the corner flow at Reynolds numbers ranging from an order of magnitude lower than the critical Reynolds number associated with the classical Blasius boundary layer. A hypothesis associated with the onset of an exponential instability via a inviscid mechanism may be proposed to explain the low transitional Reynolds number observed in experiments.

References

1. Parker J., and Balachandar, S. (1999) Viscous and inviscid instabilities of flow along a streamwise corner. Theoret. Comput. Fluid Dynamics, 13:231–270.
2. Zamir M. (1980) Similarity and stability of the laminar boundary layer in a streamwise. Proc. R. Soc. Lond. A, 377:269–288.
3. Bottaro A., Corbett, P. and Luchini, P. (2003) The effect of base flow variation on flow stability. J. Fluid Mech., 476:293–302.