# Identification and Quantification of Shear Layer Influences on the Generation of Vortex Structures

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## Summary

The analysis of complex flow fields based on the notion of organized motions of flow field features with spatial and temporal coherence, the so-called coherent structures, is one of the principal methods in the development of advanced analysis tools in fluid mechanics. For decades the focus of research was on vortex structures as the only coherent structure of interest, caused by the significant role of these structures in the understanding of the main fluid dynamical events, e.g. turbulence in boundary layers. A new topic is the extension of the flow field analysis by shear layer structures. The consideration of the shear layer structures allow additional informations in dynamical processes, e.g. generation and decay of vortex structures.

## 1 Introduction

For real-life flow control applications it is desirable to have a better understanding of fundamental fluid dynamical mechanisms and to possess methods for their automatic detection, quantification and monitoring. Traditionally, people use the concept of vortex structures, which are also known as coherent structures in the literature, to describe and generalize a multitude of fluid motions. For a viscous flow, however, shear layers are equally important. This leads to the question of their interactions, i.e. the formation of new vortices either by shear layer roll-up or by the interaction and merging of already existing vortices, and the formation of new shear layers by vortices.

Such problems have been already studied in the past, e.g. in qualitative particle visualization of flow fields to analyze vortex structures. However, without the availability of a quantitative analysis of fluid dynamics and of numerical methods for an automatic detection of relevant events.

In the present work we first identify shear layers and vortices in a given flow field using numerical implementations of analytical criteria. During their lifetime, criteria, like position, size, vorticity, enstrophy, circulation, etc. of each structure are recorded and tracked, such that a compact graphical representation of the identified dynamics can be shown. We then look for the birth of new vortices, e.g. out of shear layers or by the merging of two interacting vortices. Once these have been found "by hand" we evaluate and compare the extracted data for these events. Our intent is to find criteria for an automatic identification of events in general configurations using these methods. The current state of the work will be explained on the analysis of a two-dimensional laminar shear layer that has been generated by the merging of two parallel streams with different velocities.

## 2 Identification of Coherent Structures

### 2.1 Vortex Structures

The main vortex identification criteria are derived from the definition of a vortex structure as a finite volume of fluid particles with a rotational motion around a center line. They can be classified according to the following categories:

- The first group contains conventional criteria like vorticity, pressure, as well as the investigation of vortices using streamlines and pathlines. The inadequacies of these criteria and methods were discussed, e.g., by Jeong and Hussain [5].
- The basic concept of the second group of criteria is the eigenvalue analysis of the velocity gradient tensor $\nabla v$ in order to detect a vortical motion.
- The widely used $Q$ or Okubo-Weiss criterion, which is described in section 2.2, and $\lambda_2$ [5] criterion are based on the decomposition of the velocity gradient tensor into a rotational motion and shear stress.
- The latest group considers shear in the identification of vortex structure regions, as higher shear was identified as a critical factor in the identification of vortex structures.

One of the latest criterion, which is considering the effects of shear in the vortex identification, is the approach of Kolář [6]. The triple decomposition method of Kolář is an extension of the traditional double decomposition of the velocity-gradient tensor $\nabla u$ in to a symmetric $(\Omega)$ and an antisymmetric $(S)$ part. The motivation for the triple decomposition is the fact that vorticity cannot distinguish between pure shearing motion and vortical motion, and strain rate cannot distinguish between straining motion and shearing motion. The advantage of a triple decomposition of the velocity-gradient tensor compared to a double decomposition is the inclusion of pure shearing motion as one of the elementary motions of a fluid element.

The outcome of the triple decomposition of the velocity-gradient tensor is $\nabla u = S_{RES} + \Omega_{RES} + (\nabla u)_{SH}$ , where the term $(\nabla u)_{SH}$ represents the pure shearing motion, the term $S_{RES}$ represents the new strain-rate tensor which neglects the portion of strain rate effected by shear and the last term $\Omega_{RES}$ represents the new vorticity tensor which neglects the portion of vorticity effected by shear.

The new vortex identification criterion based on the outcome of the triple decomposition of the velocity-gradient tensor is related to the vortex identification by vorticity strength in the double decomposition. The newly determined scalar for vortex identification is called $\omega_{RES}$ and represents the *corrected* vorticity. The new criterion is Galilean invariant, it requires no thresholds and the magnitude of the scalar represents the strength of the rotational motion of a fluid element.

Although the influence of high-shear regions in the identification of vortex structure regions is a currently discussed topic in flow field analysis, the understanding of it is still not clear. Hence it is necessary to analyze the effects of shearing on vortex structure identification and vortex dynamics.

### 2.2 Shear Layer Structures

One of the main characteristics of shear-layer structures is that shear regions are also regions of high vorticity, comparable to vortex structures. Although most studies of flow-field features are based on vortex-structure analysis due to their importance for fluid dynamics, the significance of shear layers on vortex dynamics is not negligible. Shear layers are of interest, for example in vortex generation, vortex-vortex interaction, vortex-shear layer interaction, but also in the understanding of vortex structures and their identification in flow fields.

The main Eulerian shear layer identification criteria are also based on the double decomposition of the velocity gradient tensor $\nabla u$.

As shear layer regions are defined as regions of high strain, the identification of shear layers is based on the strain-rate tensor $S$. In order to identify a shear layer, it is preferable to compute a scalar value that represents the strength of the strain. One of the most important criteria is the Q criterion, i.e., the second invariant of the velocity gradient tensor, which is typically used for the identification of vortex regions, but can also be used to identify areas of high shear stress. It determines if a point of a flow field is dominated by rotation, i.e., $\|\Omega\| > \|S\| \rightarrow Q > 0$, or by strain, where $\|\Omega\| < \|S\| \rightarrow Q < 0$. Hence it follows that a point is identified as part of a shear layer if $Q < 0$, where the norm stands for the Frobenius norm of $\Omega$ and $S$ with $\|\Omega\| = \sqrt{Tr(\Omega\Omega^T)}$ and $\|S\| = \sqrt{Tr(SS^T)}$, respectively.

Another method was proposed by Haimes et al. [2], where an eigenvalue analysis of the strain-rate tensor is applied. Since $S$ is symmetric and positive, it has always three real eigenvalues $(\lambda_{S1}, \lambda_{S2}, \lambda_{S3})$. The vector formed by the eigenvalues of $S$ defines the principal axis of deformation and the norm of the second principal invariant is used as a measure of the shear. According to Haimes et al., the rate of shear stress is defined by:

$$S_H = \sqrt{\frac{(\lambda_{S1} - \lambda_{S2})^2 + (\lambda_{S1} - \lambda_{S3})^2 + (\lambda_{S2} - \lambda_{S3})^2}{6}}. \quad (1)$$

In this work, we use the criterion by Haimes et al. since the criterion works directly on the shear stress tensor and the outcome is a measure for the strength of shear stress on a fluid element.

Each of the criteria, $Q$ and $S_H$, is independent of the variation of a reference frame with constant speed; therefore, both criteria are at least Galilean invariant. Furthermore, the criterion of Haimes et al. is also rotational invariant in contrast to the $Q$ criterion, where the result depends on the orientation of the reference frame (for more detailed information see [3]). Another disadvantage of the $Q$ criterion consists of its exclusive behavior, i.e., either a region is detected as a vortex or as shear layer. Thus, the criterion decides which of both features, the vortex strength

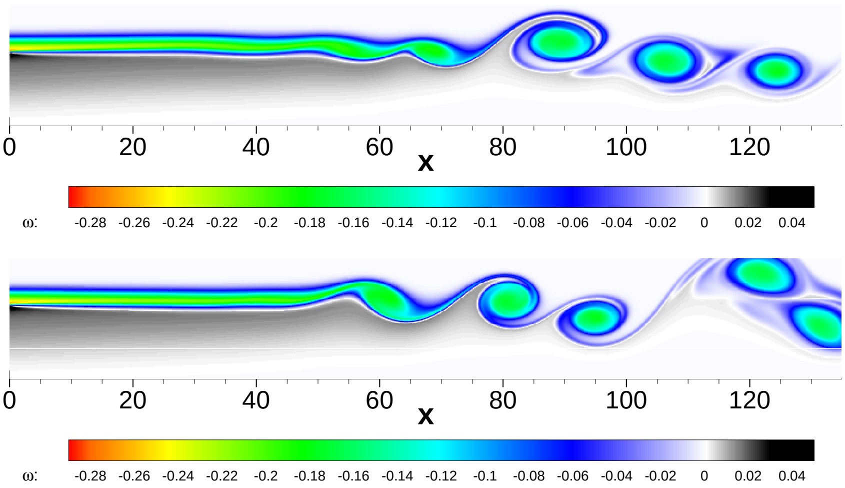


**Figure 1** Snapshots of the vorticity field of the 2D mixing layer.

or the strain, dominates at a point inside the flow. In our opinion, this makes the $Q$ criterion actually insufficient for the temporal exploration of shear layers, since they might undergo a roll-up process in order to activate the generation of new vortices. This makes it necessary to know both values, especially at points where both quantities are simultaneously present.

## 3 Identification of Vortex Generation in a Shear Layer

The present investigations are based on the results of a two-dimensional direct numerical simulation of a subsonic mixing layer behind a flat plate [1]. The analyzed case is isothermal with the Mach numbers $Ma_I = 0.8$ for the upper and $Ma_{II} = 0.2$ for the lower stream. The ratio of the streamwise velocities is $U_I/U_{II} = 4$ and the temperatures of both streams are $T = 280K$.

The wake of the plate consists of two shear layers of opposite sign, cf. figure 1. The upper one, which is stronger starts to roll-up into vortices at $x \approx 60$. Another observation is the merging of back-to-back vortex structure pairs at $x > 80$. The problem with vorticity, used in figure 1, is that it cannot distinguish between vortices and shear layer. For that purpose we use the methods introduced above, namely $\lambda_2$ for vortex and $S_H$ for shear-layer identification. According results for the first snapshot in figure 1 are shown in figures 2a and 2b, respectively.

The shear layer identification in figure 2b shows how the strong upper shear layer disappears further downstream (note the obvious contrast to the vorticity in figure 1). Inside the vortices, especially as they grow in strength, the shear develops a local minimum. This has to be expected because a solid-body rotation in the vortex cores should be shear-free. At the same time new shear is generated by friction at the

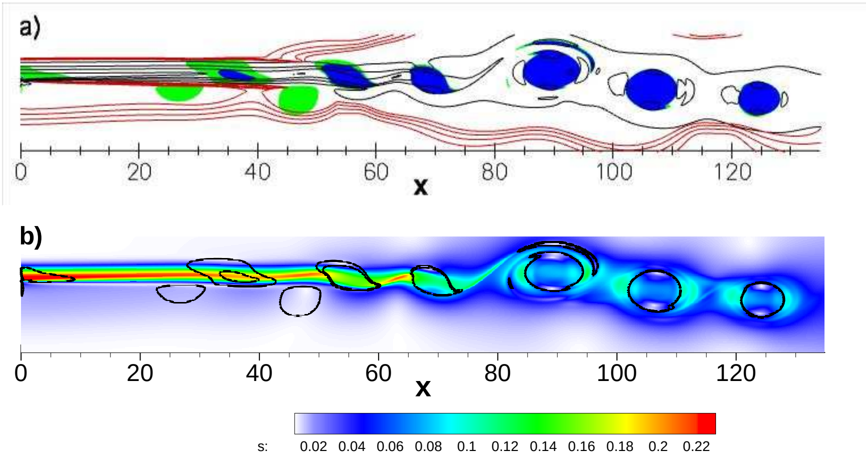


**Figure 2** Visualization of a two-dimensional flow field downstream a flat plate. (a) $\lambda_2$ (blue regions: $\lambda_2 < 0.05 * (\lambda_2)_{min}$; green regions: $0.05 * (\lambda_2)_{min} < \lambda_2 < 0.001 * (\lambda_2)_{min}$ and shear layers (black lines: $S_H$ values between $0.03\frac{1}{s}$ and $0.27\frac{1}{s}$; red lines: $S_H$ values between $0.006\frac{1}{s}$ and $0.012\frac{1}{s}$, (b) Shear layer visualization with the approach of Haimes and display of vortex structures (lines).

edges of the detected vortices and around them. This kind of behaviour has already been observed earlier in the analysis of isolated vortices [4].

The present example also reveals the dependence of the results on the chosen threshold. To show this, we have considered two different $\lambda_2$ thresholds in figure 2. However, they indicate that the identification of the strong vortices in the downstream part of the domain is independent of the chosen threshold. But the identification of the first occurrence of a vortex is strongly threshold dependent. It may happen directly at $x = 0$ or at any position further downstream, such that an objective definition is not possible.

For the purpose of a better understanding of vortex formation in the present scenario we consider the amplification of the fundamental disturbance and its higher harmonics in figure 3 (obtained from a Fourier analysis of the flow field). This decomposition allows to identify the emergence of the first structures out of very-small-amplitude instabilities. Initially, the fundamental frequency dominates as this one is the most unstable in the upstream boundary layer. As the wake of the flat plate is encountered beyond $x = 0$ the most unstable frequency shifts to a three times higher value, cf. [1]. This is why the second harmonic ($3 \cdot f_0$) grows faster there. At $x \approx 50$ all modes of the disturbance spectrum finally saturate. This coincides with the formation of the first observable local vorticity maximum within the shear layer in figure 1. The distance of the ensuing individual vortices perfectly agrees with the wave length of the according shear layer instability, i.e. the second higher harmonic. A change occurs around $x \approx 87$ where the fundamental frequency supersedes the second harmonic again. This can be associated to the vortex pairing

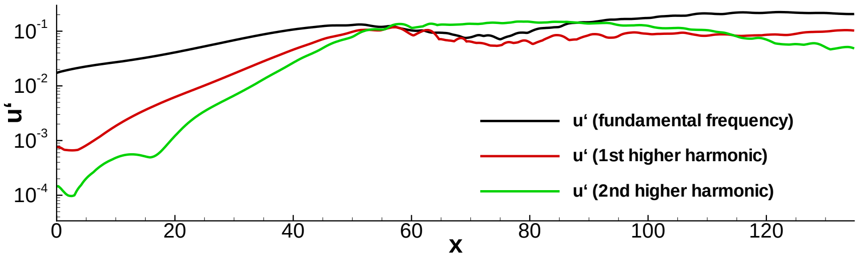


**Figure 3** Maximum amplitude of the streamwise velocity $u'$ downstream of the flat plate. The fundamental frequency and the two higher harmonics of the fundamental frequency are plotted.

between neighbouring vortices in the instantaneous pictures. Note that the fundamental herewith becomes a subharmonic of the second harmonic. The consideration of figure 3 confirms that the emergence of vortices from the free shear layer is a continuous process that starts at very low amplitudes such that the threshold-sensitivity of the vortex identification scheme is not a failure of the latter but rather a true consequence of the underlying physics.

In figures 4 and 5 we discuss the streamwise evolution of vorticity $\omega$, shear $S_H$, residual vorticity $\omega_{RES}$, and $-\lambda_2$ further. The idea is to show the interplay between vorticity, shear and vortical motion from the initial formation of vortices until the beginning of the pairing stage. For these plots the mean flow has been subtracted and the fluctuation amplitudes are shown as root-mean-square (*rms*) values. A similar analysis could be performed for instantaneous data but here we confine ourselves to the average.

Figure 4 presents vertical cuts through the data at $x = const$ while figure 5 follows the amplitudes at $y = 0$ in streamwise direction. Initially, vorticity and shear are identical and the vortex criteria remain at negligible values, which confirms our statements made further above. The first vertical cut shown in figure 4a clearly depicts the amplitude of a shear-layer instability. At the next station, the shear maximum at $y = 0$ starts to decrease with respect to vorticity. As the vortex criteria start to grow at the same time, we see that the emergence of vortices has started now. At $x = 65$ a changeover takes place: shear $S_H$ and residual vorticity $\omega_{RES}$ become equally large. This corresponds to the emergence of a clear vorticity 'blob' in figure 1 and one might think about suggesting this cross-over point as a possible unbiased criterion for the first occurrence of a vortex out of a shear layer. What follows is a dominance of the residual vorticity over shear inside the mixing layer which indicates that the latter has been split into vortices. The widening of the mixing layer in $y$-direction and the continuous decrease of the maxima further downstream is due to the averaging process when computing the *rms* over many individual vortices. A second increase of the shear maximum after $x \approx 80$ can be attributed to the generation of new shear layers that surround the individual vortices which has already been mentioned in connection with figure 2b. An additional growth occurs in-between neighbouring vortices before they merge because of the high friction there. This contributes to the increase of $S_{H,rms}$ as well.

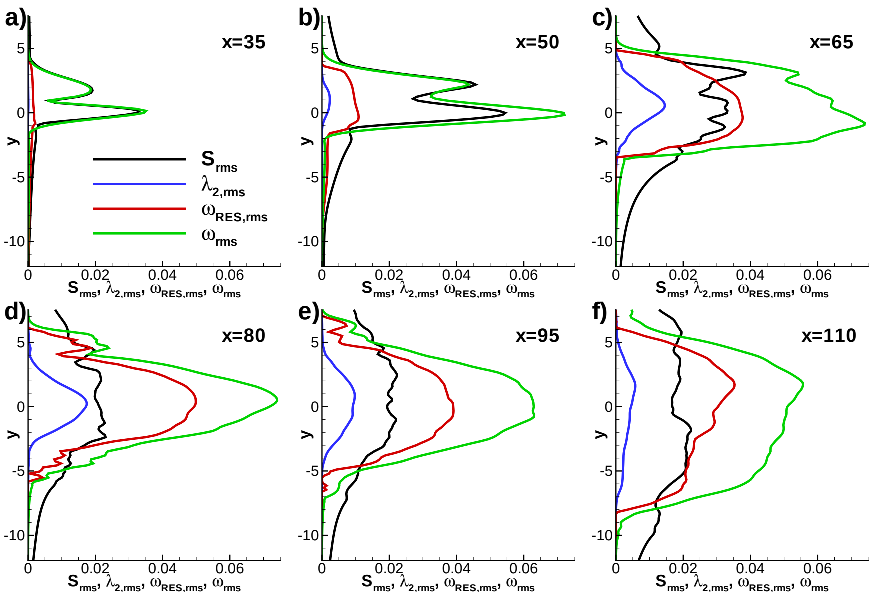


**Figure 4** Amplitudes of $\omega_{rms}$, $S_{rms}$, $\omega_{RES,rms}$ and $\lambda_{2,rms}$ at different x-positions

## 4 Conclusion

Although the focus of research was on vortex structures for decades, the discussions about the importance of shear regions in the identification of vortex structures revealed the relevance of shear in the understanding of the vortex dynamics and flow field analysis. As shown in section 2.2 it is possible to define an additional type of coherent structures, in this case the coherence is based on shear. The example in section 3 shows the importance of shear layer structures in the formation of new vortex structures, hence the mechanism is a roll-up of the shear layers caused by instabilities and the first observation of vortex structures is influenced by the chosen threshold. However, a decomposition of vorticity into pure shear and residual vorticity, which is solely due to vortical motion, yields deeper insights into the dynamics of vortex formation out of a shear layer and the further development of these vortices. Our next step will be a more detailed analysis of vortex merging using the methods presented here.

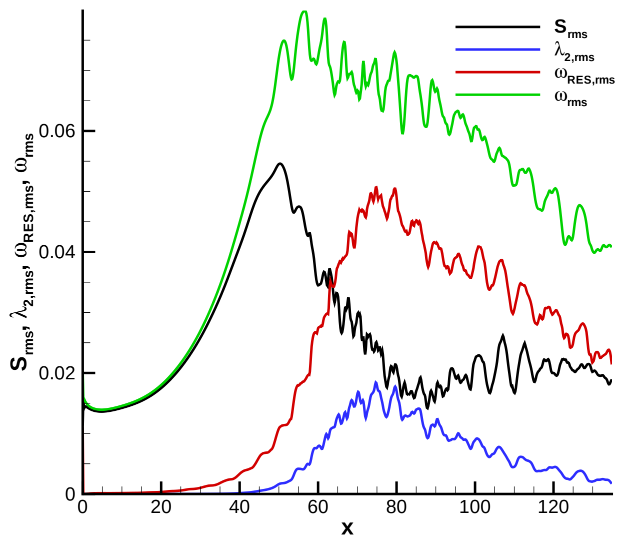


**Figure 5** Amplitudes of $\omega_{rms}$, $S_{rms}$, $\omega_{RES,rms}$ and $\lambda_{2,rms}$ at $y = 0$

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## References


[1] A. Babucke, M. Kloker, U. Rist: "Numerical Investigation of Flow-Induced Noise Generation at the Nozzle End of Jet Engines". Notes on Numerical Fluid Mechanics and Multidisciplinary Design Volume **96**, 2006, pp. 413-420.

[2] R. Haimes, D. Kenwright: "On the Velocity Gradient Tensor and Fluid Feature Extraction". In Proc. AIAA 14th Computational Fluid Dynamics Conference, 1999, pp. 3288-3297.

[3] G. Haller: "An Objective Definition of a Vortex". Journal of Fluid Mechanics Volume **525**, 2005, pp. 1-26.

[4] M. Hauser: "Untersuchungen zur Interaktion von Scherschichten und Wirbeln". Diploma Thesis, Universität Stuttgart, 2006.

[5] J. Jeong, F. Hussain: "On the Identification of a Vortex". Journal of Fluid Mechanics Volume **285**, 1995, pp. 69-94.

[6] V. Kolář: "Vortex Identification: New Requirements and Limitations". International Journal of Heat and Fluid Flow Volume **28**, 2007, pp. 638-652.