Linear Instability of Streamwise Corner Flow

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A streamwise corner is obtained when two flat plates meet at an angle to form a corner parallel to the free-stream flow direction. The interaction of the two boundary layers creates a secondary flow that makes the flow susceptible to separation and early laminar-turbulent transition, which is unwanted. Despite its simplicity, there are not many studies of this flow and current understanding of instability, laminar-turbulent transition, and turbulence is very incomplete. As such, existing computations cannot yet describe premature transition or separation observed in experiments.

On the long term the present work is intended to fill an obvious gap in predicting, understanding, and controlling laminar-turbulent transition in streamwise corners. The laminar base flow for a 90^o right-angled corner is considered. Linear stability computations have been performed using a two-dimensional local linear stability theory to compute temporal growth rates and a parabolized stability equations (PSE) approach for spatial growth. These methods have been developed out of previous work by Alizard & Robinet for two-dimensional flows [1].

Typical eigenvalue spectra for both approaches are compared in Fig. 1. The temporal amplification is for $Re = 707$ ($Re_x = 2.5 \times 10^5$) and $\alpha = 0.2$ for comparison with Parker & Balachandar [2]. The spatial case is for the frequency $\Omega = 0.08$ at $Re = 450$ based on the initial position $x_0 = 225$. All data are normalized with respect to the length scale $\delta = \sqrt{2\nu x/U_\infty}$. Different symmetries of the disturbance profiles are possible with respect to the corner bisector: "even modes" whose streamwise velocity is symmetric, and "odd modes" which are antisymmetric with respect to the bisector. On one hand this information was used to reduce the number of unknowns by a factor of 2 (by computing each class of disturbances separately) and on the other hand it was used to verify the code by solving the full problem without symmetry conditions, see Fig. 1. A grid refinement study was performed at the same time to assure the grid independence of the results.

All eigenvalue spectra exhibit a branch of eigenvalues which can be attributed to Tollmien-Schlichting (TS-) modes and an isolated corner mode.

Further insight into these is obtained from their corresponding eigenfunctions, see figures 2-4. The two most unstable eigenvalues are odd and even. They correspond to two-dimensional TS-waves of the flat-plate boundary layer. This can be seen in the eigenmodes of Fig. 2 where the amplitudes develop the shape of a TS-wave far away from the corner, both in y and z direction. Accordingly, the other TS-modes correspond to pairs of oblique waves with increasingly smaller transverse wave lengths, as illustrated by the local maxima in Fig. 3. The even-symmetric modes have a local maximum in the corner. Compared to the odd symmetries where this maximum is absent (see Fig. 2, left) they are more unstable than the odd ones, see Fig. 1.

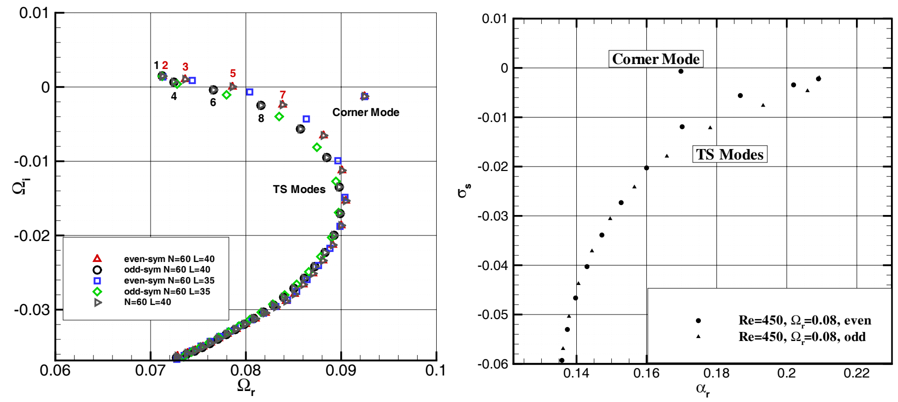

Fig. 1. Comparison of temporal (left) and spatial spectra (right)

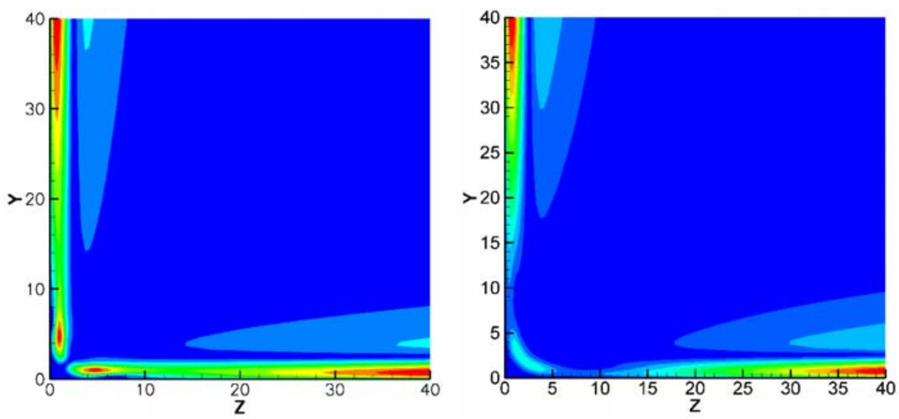

Fig. 2. Streamwise velocity component of the most unstable odd (left) and even (right) temporal TS modes at $Re = 707$ ($Re_x = 2.5 \times 10^5$) $\alpha = 0.2$

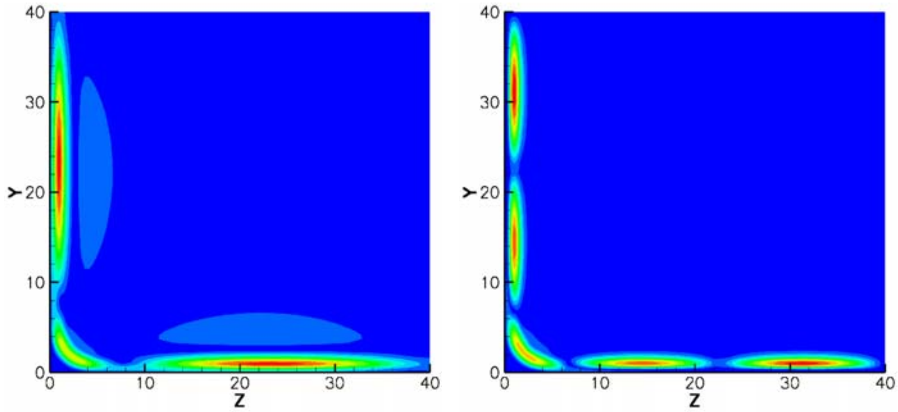

Fig. 3. Streamwise velocity contours of higher (even) TS-modes. Mode 3 (left), mode 5 (right)

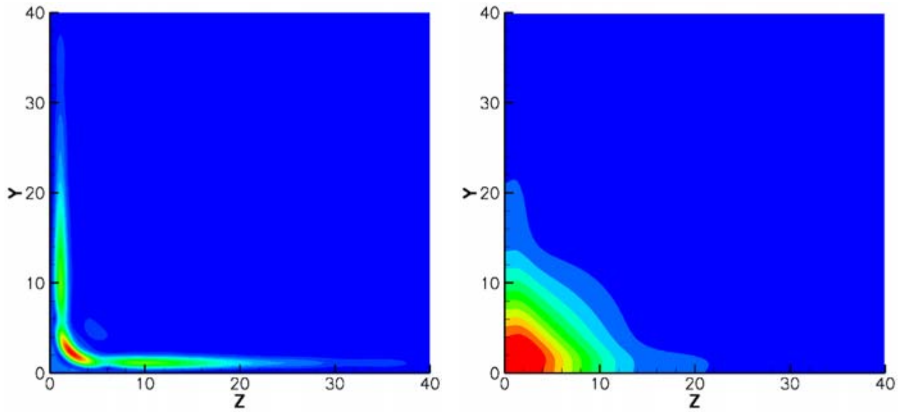

Fig. 4. Spatial structure of the temporal corner mode at $Re = 707$ ($Re_x = 2.5 \times 10^5$) $\alpha = 0.2$. Streamwise velocity component (left) and pressure (right).

The eigenfunctions are essential to classify the eigenvalues. As such, the corner mode shown in Fig. 4 is clearly distinguished by a velocity maximum which rides on the inflection point of the base-flow velocity profile in the corner bisector. Because of this it is supposed to be related to an inviscid instability. Away from the corner and along the plates the corner mode decays. Its relation to the corner is best illustrated in a plot of its pressure eigenfunction in Fig. 4, right.

Even if the corner mode is very close to the border between stable and unstable disturbances, it is never unstable in the present local stability analysis. It seems that the local theory cannot explain why experimentalists observe a premature laminar-turbulent transition of the corner flow compared to the flat-plate boundary layer. Our comparison with Parker & Balachandar [2] shows close but nor identical agreement. The latter can be attributed to slight differences in the computed base flow, which is very sensitive, cf. Ridha [3]. This aspect is now further investigated in the work of Alizard *et al.* [4].

However, extending our computations to a full PSE that follows a given frequency in downstream direction such that non-parallel growth of the flow is no longer neglected, we made an unexpected observation, shown in Fig. 5. Non-parallel effects are irrelevant for the TS modes as in the case of the Blasius boundary layer, but the corner mode which was stable before now becomes unstable as well. Different non-parallel criteria will be studied in order to determine precisely the critical Reynolds number associated with the corner mode. This discovery could help to explain experimental results.

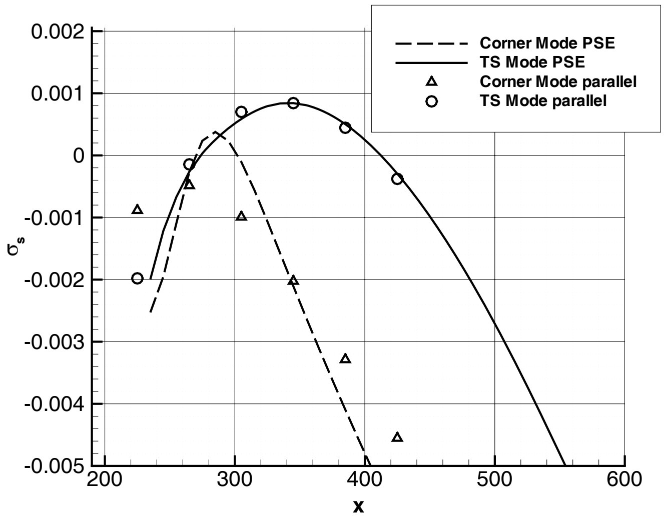

Fig. 5. Comparison of parallel theory with PSE analysis for $\Omega = 0.08$

References

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