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## ACTIVE CONTROL OF NONLINEAR DISTURBANCES IN A BLASIUS BOUNDARY LAYER

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### ABSTRACT

Using the tools of Direct Numerical Simulation (DNS) and Linear Stability Theory (LST) we are studying active concepts to control nonlinear stages of transition. Besides the investigation of the well known wave superposition approach we developed a new method to actively control transitional disturbances. This concept uses the feedback of instantaneous flow data (wall shear stress or spanwise vorticity), obtainable at the wall to drive plain actuators. Avoiding long propagation distances between sensor and actuator this procedure (called  $\omega_z$ -control) results in a very effective damping in linear and nonlinear cases. With some extra improvements like spatial filters between sensor and actuator it is possible to delay transition even in strongly nonlinear scenarios like the K-breakdown scenario (transition due to fundamental resonance).

Both methods have been investigated in a Blasius boundary layer. Besides the K-breakdown already mentioned for basic investigations a "linear" scenario was used, consisting of Tollmien-Schlichting waves with different propagation angles.

### NOMENCLATURE

$x, y, z$  Spatial coordinates ( $y$ : wall normal)  
 $u, v, w$  Velocities in  $x$ -,  $y$ - and  $z$ -direction  
 $h$  Spatial FIR-filter vector  
 $A$  Amplitude factor between sensor- and actuator-signal  
 $H$  linear transfer function, Fourier transformed  $h$   
 $\alpha$  Spatial wavenumber

$\beta$  Frequency

$\omega_x, \omega_y, \omega_z$  Vorticity in  $x$ -,  $y$ - and  $z$ -direction

$\Theta$  Phase between sensor- and actuator-signal

### INTRODUCTION

So far, mainly passive methods such as smooth surfaces or advantageous pressure distributions have been used to reduce aerodynamic drag of wings, shifting the boundary layer transition downstream. Unfortunately, beyond a certain Reynolds number these approaches don't work in a satisfactory manner. In this case, approaches which actively damp disturbances in boundary layers offer some new possibilities.

In this context some active approaches like the superposition of disturbances with opposite phase to the initial disturbances have been developed. Here, disturbances with the same amplitude but opposite phase are superimposed to the initial disturbance. The (linear) addition of both modes results in the reduction or cancellation of the initial perturbation.

The superposition method works very well especially in combination with adaptive FIR-filters for linear disturbances (e.g. with very small amplitude) (Baumann et al., 1998) but once the amplitude of the initial disturbance exceeds a certain level resulting in nonlinear interactions between the modes, this linear approach fails and other concepts, less influenced by nonlinearities, have to be taken into account.

In contrast to approaches based on optimal control theory which yield optimal results for a specific case, we follow a path

which can be more or less directly implemented into application using the direct feedback of instantaneous signals from the flow field (Gmelin and Rist, 2000). Our approach, better suited for nonlinear disturbances is the “ $\omega_z$ -control”. In this case the spanwise vorticity at the wall is multiplied by a gain  $A$  and prescribed as a  $v$ -boundary-condition on the wall (blowing/suction) with a certain phase shift  $\Theta$  (Fig. 1).

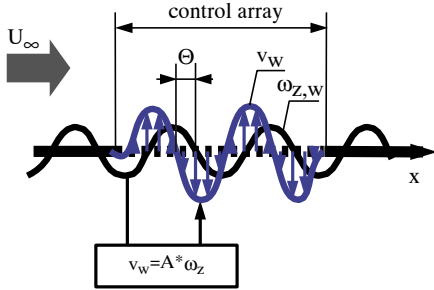


Figure 1. CONTROL PRINCIPLE OF THE  $\omega_z$ -CONTROL.

All investigations (LST and DNS) have been performed in a Blasius boundary layer with a free stream velocity of  $U_\infty = 30\text{m/s}$ . The Reynolds number calculated with the displacement thickness has a value of  $Re_{\delta_1} \approx 500$  at the beginning of the observed domain ( $x_0 = 0.8443$ ) and a value of  $Re_{\delta_1} \approx 1300$  at the end. All length scales are made dimensionless with  $L = 0.05m$ .

## RESULTS OF THE LINEAR STABILITY THEORY (LST)

To study the behaviour of linear instability waves under the influence of active  $\omega_z$ -control using LST the boundary conditions at the wall for the Orr-Sommerfeld equation (and the Squire equation) had to be changed (index  $w$  denotes wall properties) to

$$v_w = A \cdot \omega_{z,w} \quad (1)$$

with  $A = |A| \cdot e^{i\Theta}$ ,

where  $|A|$  is the amplitude factor and  $\Theta$  is the phase difference between  $v_w$  and  $\omega_{z,w}$ . Due to the ability to express  $\omega_z$  in terms of  $u$  and  $v$  the existing eigenvalue problem remains homogeneous. The resulting eigenvalues and eigenfunctions show very good agreement to the results obtained using DNS. Fig. 2 shows the comparison between  $u$  and  $v$  disturbance amplitude profiles obtained from DNS and LST under the influence of the changed boundary conditions. The difference which is nevertheless visible is due to nonparallel effects which are not considered in the LST calculations.

Looking on Fig. 3 the influence of active control on the unstable region of the Blasius boundary layer flow can be observed.

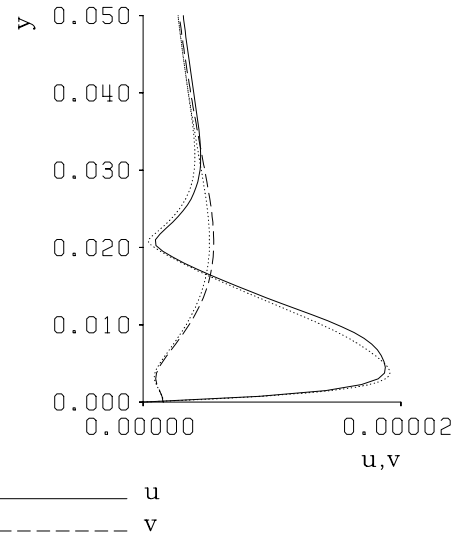


Figure 2. COMPARISON BETWEEN  $u$  AND  $v$  DISTURBANCE AMPLITUDE PROFILES OBTAINED FROM DNS (SOLID AND DASHED LINES) AND LST (DOTTED LINES). LINEAR CASE WITH  $|A| = 0.0001$  AND  $\theta = \pi/2$  AT  $x = 3.46$ . (QUANTITIES ARE NORMALISED WITH  $u_\infty = 30\frac{m}{s}$  AND  $L = 0.05m$ ).

Already small amplitudes are sufficient for a strong damping effect. Thus, for an amplitude factor  $|A|$  larger than approximately  $5 \cdot 10^{-5}$  and for a phase angle of  $\Theta = 0$ , the boundary layer is stable for all considered frequencies and downstream positions. Detailed investigations concerning the dependence of the eigenvalues on the phase angle are illustrated in Fig. 4. Amplification rate  $\alpha_i$  and streamwise wavenumber  $\alpha_r$  change in an almost sinusoidal way with  $\Theta$ . Thus, with the right phase angle for a given gain  $A$  a broad range of desired amplification can be reproduced. To achieve maximum performance in terms of attenuation the phase shift  $\Theta$  has to be adjusted between  $\pi/4$  and  $\pi/2$ .

## DIRECT NUMERICAL SIMULATION (DNS)

To investigate the behaviour of nonlinear waves in the Blasius boundary layer and to verify the LST results in the linear case, a number of calculations using DNS were carried out. All simulations were performed in a rectangular integration domain with the spatial DNS-code (Rist and Fasel, 1995) already used for preliminary results in earlier publications (Gmelin et al., 1998). The flow is split into a steady 2D-part (Blasius base flow) and an unsteady 3D-part. The  $x$ -(streamwise) and  $y$ -(wall-normal) directions are discretized with finite differences of fourth-order accuracy and in the spanwise direction  $z$  a spectral Fourier representation is applied. Time integration is performed by a classical fourth-order Runge-Kutta scheme.

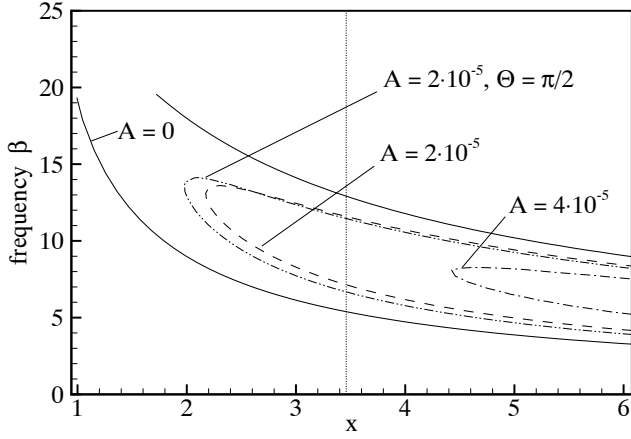


Figure 3. CURVES OF ZERO AMPLIFICATION FOR THE BLASIUS BOUNDARY LAYER FLOW WITH DIFFERENT GAINS  $A$  AND PHASE ANGLES  $\Theta$ . FREE STREAM VELOCITY WAS  $U_\infty = 30 \frac{m}{s}$  AND  $x$  WAS MADE DIMENSIONLESS WITH  $L = 0.05m$ . THE DOTTED LINE MARKS THE POSITION IN THE  $x/\beta$ -DIAGRAM OF THE CALCULATIONS SHOWN IN FIG. 4

### NONLINEAR 2D-MODES

We were able to show the excellent performance of the new approach for linear waves and a very good agreement to results of the linear stability theory (Gmelin and Rist, 2000). For nonlinear cases (i.e. cases with high amplitudes, where modes cannot evolve independently) we found a good attenuation of the fundamental modes, but at a certain amount of control high frequency modes (higher harmonics and nonlinearly generated waves) were amplified strongly and let the fundamental modes rise again. Looking at Fig. 4 b) one can observe that the optimal phase shift between  $\omega_z$  and  $v$  is more or less independent of the frequency. Thus, controlling with a fixed time delay (corresponding to the phase  $\Theta$  in our LST calculations) between sensor and actuator signal yields for different frequencies to different, non optimal control phases, an effect which can be observed in the DNS-results shown in Fig. 5. The solid lines in Fig. 5 show the  $u_{max}$ -amplitudes vs.  $x$  of a nonlinear pure 2D simulation where  $\omega_z$ -control is applied with a fixed time delay between sensor and actuator signal. This delay corresponds to a phase shift of  $\Theta = \pi/2$  for the fundamental mode (1,0) (the first index denotes multiples of the disturbance frequency  $\beta = 10$ , the second multiples of the basic spanwise wave number  $\gamma = 20$ ) which is desirable but to a phase shift of approximately  $\Theta = \pi$  for the first higher harmonic mode with  $\beta = 20$  (undesirable). Besides a proper damping of the fundamental mode (1,0) it can be seen that the first higher harmonic mode is damped worse. Compared to the amplification rate of the uncontrolled mode calculated with LST (single line in

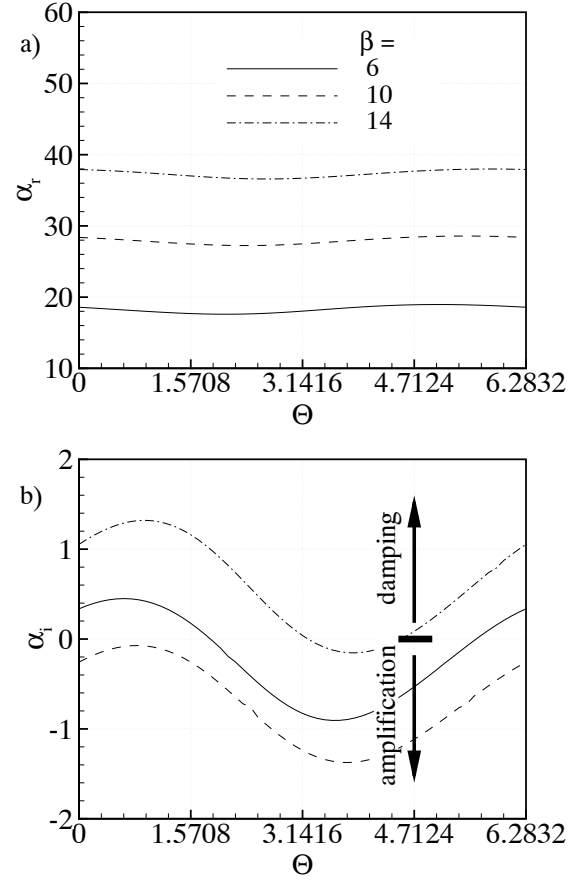


Figure 4. VARIATION OF THE EIGENVALUES WITH RESPECT TO THE PHASE ANGLE  $\Theta$  BETWEEN  $\omega_{z,w}$  AND  $v_w$  FOR  $|A| = 1 \cdot 10^{-4}$  AT THE POSITION MARKED IN FIG. 3 FOR DIFFERENT FREQUENCIES.

Fig. 5), the mode (2,0) is less attenuated, an effect which is not in accordance with the goal of control. To avoid this disadvantageous behaviour it is necessary to prescribe the desired phase shift dependent on the streamwise wavenumber resp. frequency. Therefore we applied a spatial *FIR-Filter* of length  $l$  (Fig. 6) to the input data.  $l$  samples in  $x$ -direction are multiplied with the filter-vector  $h$  to get the wall-normal velocity  $v$  at the wall:

$$v_w(x, t) = |A| \sum_{x'=x-l/2}^{x+l/2+1} \underbrace{h(x' - x + \frac{l}{2} + 1)}_{i(\text{Fig. 6})} \cdot \omega_{z,w}(x', t) \quad (2)$$

Transformed in Fourier space via the convolution theorem we obtain:

$$V_w(x, \alpha) = A \cdot H(\alpha) \cdot \Omega_{z,w}(x, \alpha) \quad (3)$$

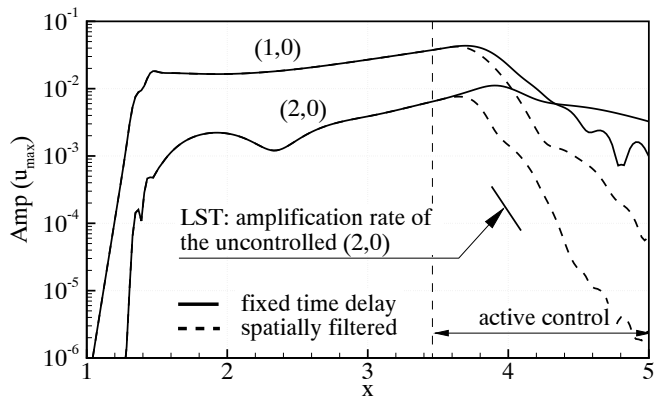


Figure 5.  $u_{max}$ -AMPLITUDES OF A NONLINEAR 2D-MODE (1,0) WITH ACTIVE CONTROL VIA  $\omega_z$ -FEEDBACK. DUE TO THE HIGH AMPLITUDE A HIGHER HARMONIC MODE (2,0) IS GENERATED.

where  $V_w$ ,  $H$  and  $\Omega_{z,w}$  are the transformed  $v_w$ ,  $h$  and  $\omega_{z,w}$  respectively. This corresponds to a multiplication of the input signal with a complex transfer function to obtain the output  $V_w$ -signal. Thus, it is possible to filter the input data with respect to their spatial wavenumber and to choose the optimal phase relation for every mode. An additional desired effect is the prevention of instabilities, which might be introduced unintentionally by the actuator response to the flow field. In Fig. 5 the effect of this spatial filter on the damping capabilities of the  $\omega_z$ -approach can be seen clearly (dashed lines). Besides a better damping of the fundamental mode, the first higher harmonic (2,0) is affected in a proper way and is attenuated as well.

### NONLINEAR 3D-DISTURBANCES

As a test case for the effect of the  $\omega_z$ -approach on disturbances with large 3D-amplitudes, a typical K-breakdown scenario (Fig. 7) is investigated where a fundamental 2D mode (1,0) with large amplitude and a steady disturbance (0,1) are excited initially. Because of nonlinear interactions the 3D-mode (1,1) is generated and falls in resonance with the fundamental 2D-mode. The other 3D modes arise due to nonlinear combinations. When the strongly amplified 3D-waves have reached the amplitude level of the fundamental mode, saturation can be observed and transition to turbulence takes place (dotted lines) downstream of  $x \approx 4.3$ .

Simultaneously with the occurrence of resonances we can see the formation of  $\Lambda$ -vortices (Fig. 8 a)) leading to typical "spikes" in the local velocity-signal. These transitional structures collapse rapidly and lead to the formation of turbulent spots by generation of smaller scale structures.

Applying  $\omega_z$ -control (with the spatial FIR-filter described above) to the K-breakdown scenario in a very late, nonlinear

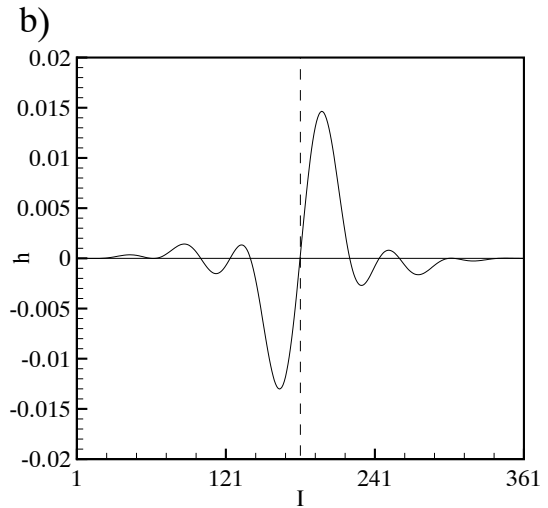
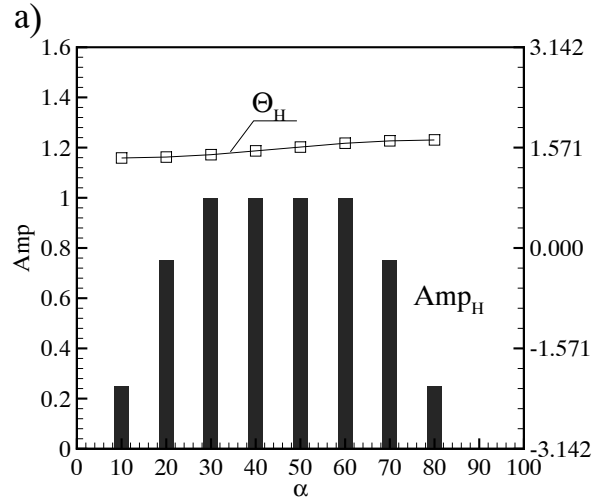


Figure 6. TRANSFER FUNCTION AND FILTER COEFFICIENTS OF THE SPATIAL FIR-FILTER USED TO STABILISE THE  $\omega_z$ -CONTROL.

stage (Fig. 7) two main control effects can be distinguished: first, the direct damping of nonlinear disturbances and secondly, the disruption of the resonant behaviour. The first effect is comparable to a linear  $\omega_z$ -control case where it is possible to directly damp TS-disturbances, the second effect results from the modified wave speed of the resonant modes which are 'detuned' due to the altered wave number (Fig. 4) under the influence of control (Gmelin et al., 2001).

The formation of  $\Lambda$ -vortices followed by a rapid collapse in the uncontrolled case consequently can be prevented in the controlled case (Fig. 8 b)) and transition can be shifted far downstream.

Unsteady modes are damped very efficiently but steady disturbances (modes (0,1), (0,2)) are hardly influenced by the con-

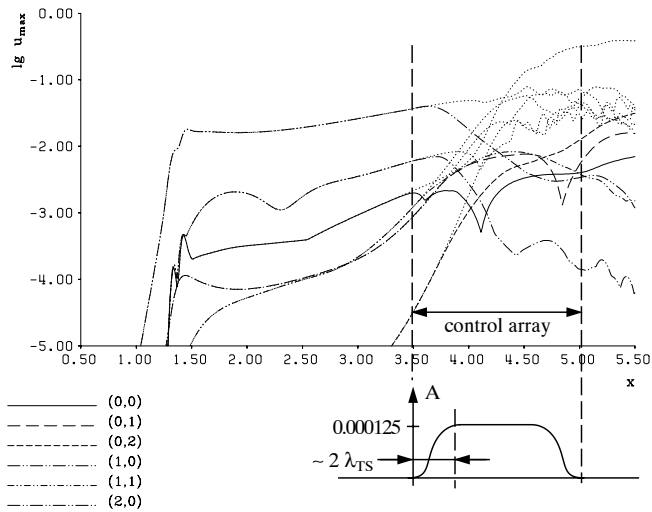


Figure 7. K-BREAKDOWN,  $u_{max}$ -AMPLITUDES VS.  $x$ . MODES  $(h,0)$  AND  $(h,1)$  CONTROLLED ( $|A| = 1.25 \cdot 10^{-4}$ ,  $\Theta \approx \frac{\pi}{2}$ ). DOTTED LINES: UNCONTROLLED CASE. ONLY THE MOST IMPORTANT MODES ARE SHOWN HERE. SMALL PICTURE: SPATIAL DISTRIBUTION OF THE CONTROL GAIN  $|A|$  WITH A SINE-LIKE RAMP FUNCTION ON BOTH SIDES.

trol and decay only because of the loss of energy-transfer from the unsteady modes. From Fig. 9 it can be seen that after approximately ten fundamental time periods of control the unsteady parts of the disturbances have already vanished whereas the remaining streak-like structures are convected downstream very slowly.

To compare the feedback-approach with "traditional" superposition techniques in Fig. 10 the  $u_{max}$ -amplitudes of the fundamental mode  $(1,0)$  of six different simulations are shown. Besides the uncontrolled case (dotted) and the simulation with  $\omega_z$ -control applied (solid) four cases can be observed, where control was applied via superposition of antiphase disturbances (dashed). These simulations show that the application of the wave superposition principle at a late stage of transition (Fig. 10) results in a negligible damping effect due to nonlinear interactions between the occurring modes which were not affected by the linear approach. In late stages of transition  $\omega_z$ -control clearly outperforms wave superposition approaches.

## CONCLUSIONS

With the aid of Direct Numerical Simulations (DNS) it was possible to develop a simple, yet effective control algorithm to actively control the laminar-turbulent transition occurring in a 2D boundary layer. It combines two main effects: the direct attenuation caused by a change of the energy properties and a reduced resonance according to an altered phase velocity of the

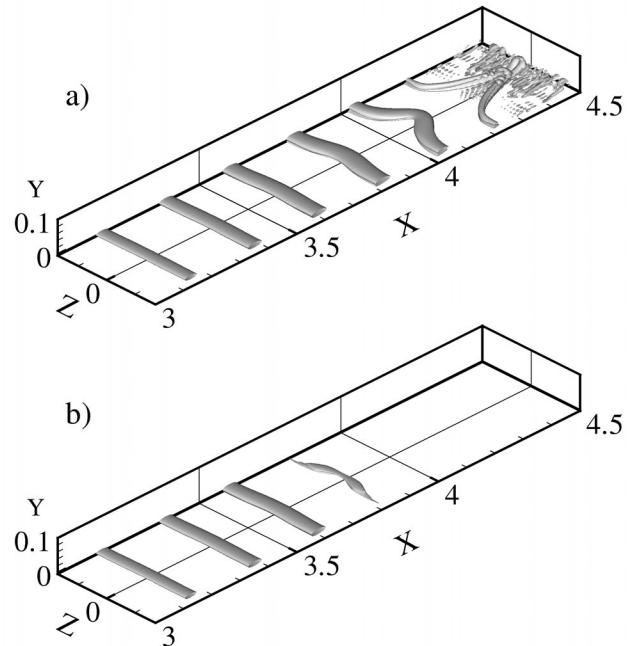


Figure 8. VISUALISATION OF VORTEX-STRUCTURES FOR THE UNCONTROLLED (a) AND CONTROLLED (b) CASE WITH AID OF THE  $\lambda_2$ -CRITERIUM OF JEONG AND HUSSAIN (Jeong and Hussain, 1995).

involved modes. Calculations using LST show a strong dependence of the resulting wave number and amplification rate on the chosen amplitude and phase difference between  $\omega'_z$  (sensed) and  $v'_{wall}$  (stimulated).

It is shown that this approach is superior to wave superposition methods especially close to transition where the boundary layer instabilities have reached a highly nonlinear stage. Further investigations have to show how far transition can be shifted downstream and whether a complete relaminarisation of the flow is possible using this approach. Also strategies to reduce steady three-dimensional (streak-like) disturbances to return the flow to its undisturbed state would be highly welcome.

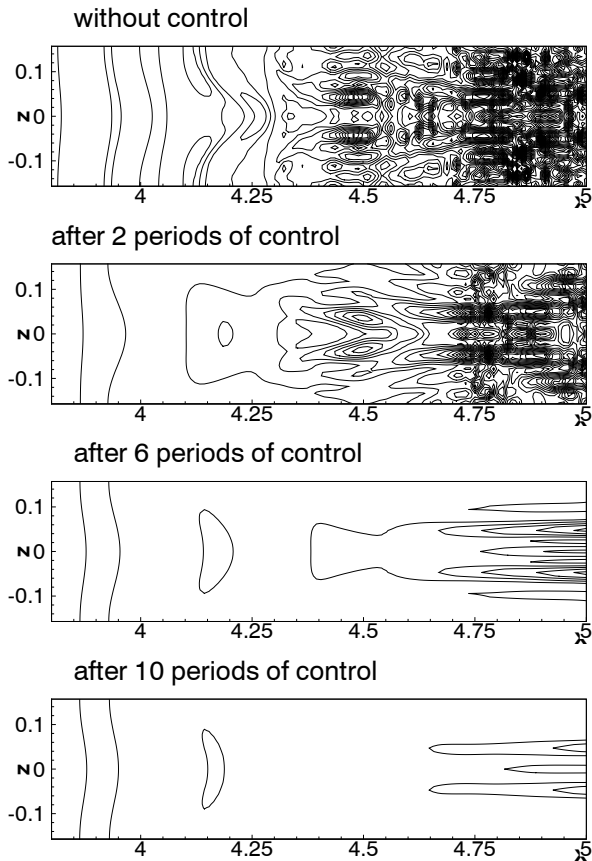


Figure 9. CONTOURS OF SPANWISE VORTICITY AT THE WALL FOR THE K-BREAKDOWN SCENARIO WITH AND WITHOUT  $\omega_z$ -CONTROL AT DIFFERENT TIME STEPS.  $\omega_z/\sqrt{Re} = -0.5 \dots 2.5$ ,  $\Delta\omega_z/\sqrt{Re} = 0.1$ ,  $Re = 100000$ .

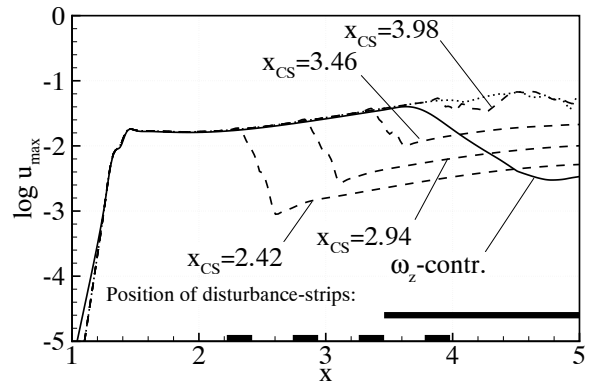


Figure 10. K-BREAKDOWN SCENARIO, MODES (1,0) CONTROLLED VIA SUPERPOSITION OF ANTI-Phase DISTURBANCES AT FOUR DIFFERENT LOCATIONS COMPARED TO THE  $\omega_z$ -APPROACH AND THE UNDISTURBED CASE ( $x_{CS}$  = POSITION OF THE CONTROL STRIP).

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