UNSTEADY DISTURBANCE GENERATION AND AMPLIFICATION IN THE BOUNDARY-LAYER FLOW BEHIND A MEDIUM-SIZED ROUGHNESS ELEMENT

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Abstract: In the present work we investigate receptivity and disturbance amplification behind a circular roughness element in a zero-pressure-gradient flat-plate boundary layer with the aim to identify and understand the basic mechanisms at work. The low disturbance background and the high repeatability of the direct numerical simulations allow to evaluate the different contributions of acoustic-roughness receptivity and of local disturbance amplification. A distinct feature of the boundary-layer flow with roughness element are streamwise streaks which develop from the spanwise edges of the roughness. Small-scale three-dimensional disturbances develop and amplify within these streaks both by instability and by receptivity. For the present small amplitude unsteady forcing however, they remain confined to these areas. In contrast to this, the far-field receptivity result is similar to a wave train generated by periodic suction and blowing at the wall.

Key words: Direct numerical simulation, receptivity, roughness, Tollmien-Schlichting wave

1. INTRODUCTION

In the past several years many investigations of the different routes to transition in generic boundary layer flows have been performed and tools have been developed that could predict the location of laminar-turbulent transition if the initial amplitudes were known accurately enough. In conclusion of the German priority research program on laminar-turbulent transition (cf. Wagner et al., 2003) we may say that finding the relevant initial disturbances is *the* key issue for a successful transition prediction in the future. In addition, there is considerable interest in studying all aspects of roughness-induced (bypass) transition since this is a significant feature in aerodynamics at medium Reynolds numbers.

2. PROBLEM FORMULATION

With the present work we want to contribute to a better understanding of the initial stages of laminar-turbulent transition behind an isolated roughness element with a circular plan form which is embedded in a two-dimensional incompressible flat-plate Blasius boundary layer. The roughness element is placed at $Re^*=855$, where Re^* is the Reynolds number based on the boundary layer displacement thickness δ^*. Here, x, y, and z denote the streamwise, wall-normal, and the spanwise coordinates, respectively. The contour of the roughness element is defined by

$$h(x, z) = h_0 \cos^3(\pi r/2b),$$

with height $h_0 = 0.5\ \delta^*$, radial coordinate r, and radius $b = 8.42\ \delta^*$. Since the height of the roughness amounts to *50 %* of the local displacement thickness of the unperturbed boundary layer, the present case cannot be treated by (linear) receptivity theory but our data may be used for future comparisons with non-linear theories or experiments.

In the absence of external disturbances a steady three-dimensional base flow develops. This is computed using the numerical method developed by Rist and Fasel (1995) augmented by the “immersed boundary technique” of Peskin (1977) to provide an efficient modeling of the roughness element without using body-fitted coordinates. Taking the latter would have increased the memory and computer time of the computations in a drastic manner.

Two controlled disturbance scenarios are then computed and compared to each other. One with a harmonic modulation of the inflow profile with the so-called second Stokes’ problem which mimics the presence of a two-dimensional acoustical wave in the free stream, such that an acoustic-wall-roughness type of receptivity interaction takes place at the roughness element. In a second case time-wise harmonic perturbations are introduced by suction and blowing at the wall upstream of the roughness element in order to force a two-dimensional Tollmien-Schlichting wave that travels over the roughness element such that changes of the stability behavior become apparent. Each of the two cases has been computed for base flows with and without roughness elements to further clarify the differences. More information can be found in Wörner (2004). Linear stability calculations (LST) based on local and averaged velocity profiles are then used to further clarify the mechanisms.

3. RESULTS

The roughness element leads to several interesting three-dimensional features within the steady base flow which are illustrated in Figure 1. Streamwise parallel longitudinal streaks of low- and high-speed fluid develop at the spanwise corners of the roughness element. These appear as bright and dark vertical stripes in the *u*-velocity contours at the last three *x=const* planes in the figure. They persist for a long distance downstream of the roughness and their spanwise gradients needed an unexpectedly high spanwise resolution of the computations (*80* Fourier modes with their complex conjugates, verified with a simulation run with twice as much). The streaks are accompanied by weak streamwise vorticity only, such that streamlines which are shown for illustration are only mildly deformed. Still, the identified vortex system resembles that found for much larger obstacles which protrude the boundary layer, like a pair of horseshoe vortices.

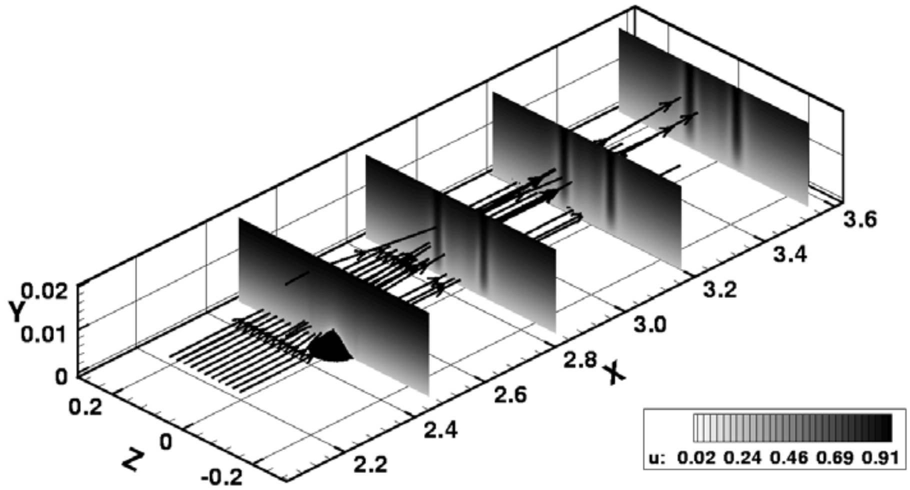

Figure 1. Illustration of the base flow using streamlines and contours of the streamwise velocity component at constant downstream positions.

The unsteady results shown in the following figures were obtained by first taking the difference of two according simulations, one with roughness element and the other one without, and then by computing the second time derivative according to Maucher et al. (1997). This helped to reduce remaining transients that were still present in the unsteady simulation results with roughness element.

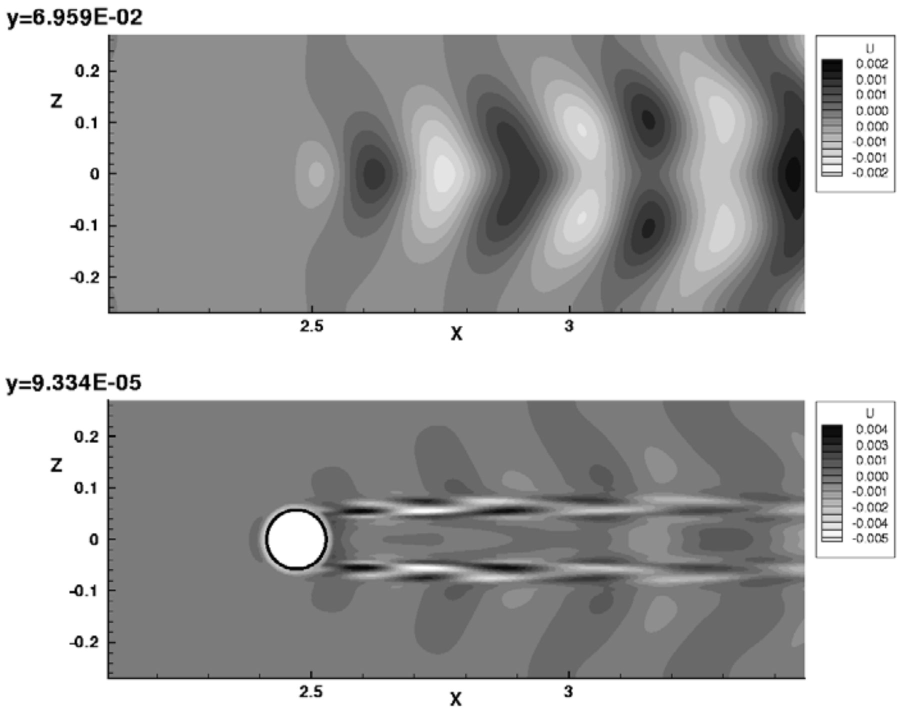

Figure 2. Disturbance field created by interaction of a periodic free-stream perturbation with the roughness element; top: far away from the wall ($y = 8.14\ \delta^*$), bottom: close to the wall ($y = 0.011\ \delta^*$).

The addition of a two-dimensional sound wave to the free stream has a two-fold effect on the disturbance field. Far away from the wall (Fig. 2 top) a characteristic wave-train develops that has already been observed earlier when the boundary layer has been forced by a harmonic point-source, e.g. by Stemmer & Kloker (2000). Note the characteristic development of two spanwise maxima downstream of the source which is due to superposition of several oblique Tollmien–Schlichting waves of moderate obliqueness angles. However, close to the wall (Fig. 2 bottom) highly oblique waves that appear in the streamwise streak regions dominate over the wave-train part of the disturbance.

A comparison of DNS disturbance profiles with eigenfunctions of linear stability theory (LST) is shown in Fig. 3. The LST computations are based on mean-flow profiles extracted from the DNS at $x = 2.7$. Two different LST-analyses have been made, one for the spanwise-averaged, the other one for the local profile extracted in the middle of the high-speed streak. In the first case the disturbance maximum is placed further away from the wall compared to the second. The figure proves that velocity fluctuations with small spanwise wave numbers in the DNS behave like eigenmodes of the averaged flow, while those with large wave numbers belong to eigen-fluctuations of the streak profile. The first correspond to the harmonic wave train seen in the far field while the latter make up the highly oblique structures in the streaks. Since the stability characteristics of the base flow with roughness element play a major role for both of them, another case where a two-dimensional Tollmien–Schlichting wave travels over the roughness element will be considered next.

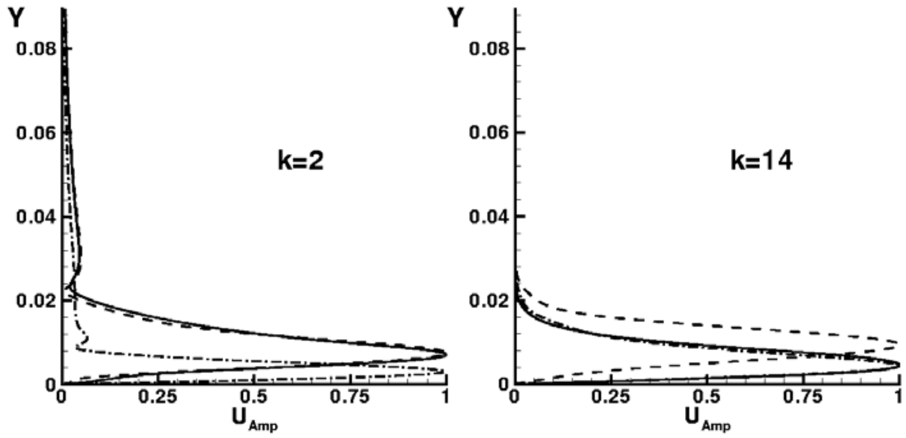

Figure 3. Comparison of two DNS disturbance profiles with eigenfunctions of linear stability theory based on the spanwise averaged velocity profile (– –) and on the streak profile (– · –), k = spanwise wave number.

Figure 4 presents instantaneous disturbance structures of this second case which can be directly compared with the first case in Fig. 2 (bottom). Again, the disturbance is composed of a wave train and dominant local structures in the streak. Compared to above, they have less phase shifts in spanwise directions but they consist of eigenmodes of the profile from the high-speed streak, while disturbances which make up the wave train correspond to eigenmodes of the averaged velocity profile, as before.

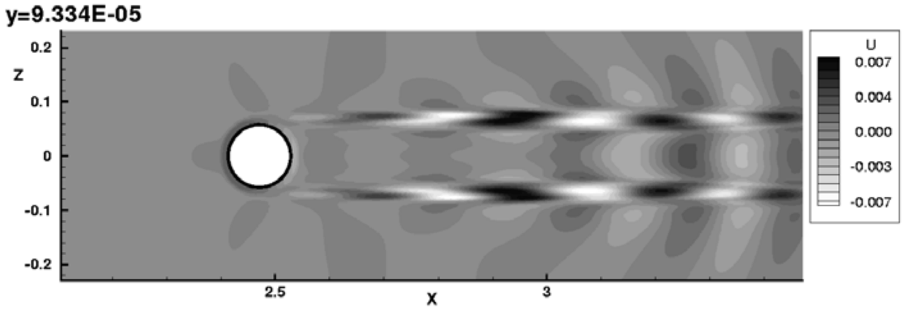

Figure 4. Disturbance field close to the wall induced by interaction of a TS-wave with the roughness element.

4. CONCLUSIONS

An isolated roughness element in a flat-plate boundary layer that interacts with an impinging two-dimensional sound or Tollmien–Schlichting wave was found to scatter two kinds of oblique waves. Close to the wall, highly oblique disturbances dominate over the wave train part. For the present small disturbance amplitudes these highly oblique waves remained confined to the streaks, but for larger amplitudes they may lead to bypass transition.

5. REFERENCES

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