K. BAYSAL, T. SCHAFHITZEL, T. ERTL, U. RIST: Extraction and Visualization of Flow Features

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**Abstract** This paper presents tools which allow an advanced investigation of spatial and temporal progress of segmented vortex structures in flow fields with the consideration of vortex dynamics. The work is based on the local and Eulerian detection and segmentation of vortex structures and the visualization of these structures with recent visualization methods, e.g. the application of LIC-textures on the surfaces of 3d structures. The second step is the tracking of vortex structures based on the time-dependent integration of vortex core lines. In the third step, according to the vortex dynamics, the individual degree of impact is determined by computation of induced velocities via the Biot-Savart equation for each vortex. The induced velocities are used to determine the induced kinetic energy and enstrophy, which allows the evaluation of a global value of influence for each vortex structure, that can be tracked in time.

#### **1** Introduction

The physical understanding of boundary layers or mixing layers is of fundamental interest for fluid dynamical research. A principal method in the investigations of physical aspects of flow fields is the observation of characteristic features of the flow fields, which are coherent in space and time. Although the so-called coherent structures or vortex structures are well-known in the analysis of flow fields, e.g. laminar-turbulent transition or effects of flow control mechanisms, the investigations of real-life flow fields based on coherent structures are not straightforward.

The aim of this paper is to present techniques for an advanced investigation of three-dimensional and time-dependent flow fields. The first goal of this work is the

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automatic identification and segmentation of vortex structures in order to find an appropriate visualization, which allows the engineer to investigate the extracted features in an interactive manner. For the visualization, recent algorithms have been applied, which exploit newest graphics hardware features. This is followed by vortex tracking, i.e., the time-dependent mapping of individual vortex structures to capture their temporal development. For this purpose, a framework has been developed, which provides, beside the investigation of vortices in a single snapshot, also the time-dependent visualization of these structures. This allows an advanced analysis of kinematic properties of the vortex structures, e.g. size, shape, and vorticity strength. The results of these studies allow the acquisition of quantitative information for flow fields, which are necessary in the development of flow field models or in the comparison and validation of experimental and numerical studies.

The second goal is the consideration of vortex dynamical effects, which allows the study of interactions between vortex structures, e.g. vortex merging, and selfexcited effects, e.g. the autogeneration of hairpin vortices, which is a significant mechanism in the generation of turbulence. The strategy of this work is the determination of the velocity field induced by each vortex structure in a flow field and their influence in the time-dependent development of other vortex structures. In order to support the consideration of vortex structure, which is then used to determine the time as well as the vortex structure which perturbs the selected vortex structures enables the investigation of effectiveness of flow-control mechanisms.

## 2 Flow-Features Identification

The main vortex identification methods are based on the widely accepted definition of a vortex structure as a finite volume of fluid particles with a vortical motion around a center line. The complexity of the vortex structures and lacks in the understanding of these structures caused discussions of different aspects of vortex structures and their identification. This resulted in several approaches for vortex identification, whereby the focus was for the last few years on the influence of axial stretching and shearing [3] on vortex identification, as investigations of vortex regions with high shear had revealed a gap in the reliability of the widely used criteria, e.g.  $\lambda_2$  [2] or the Okubo-Weiss criterion, also known as Q criterion.

Although the influence of high-shear regions in the identification of vortex structure regions is in the focus of discussions, the understanding of it is still not clear. Hence it is necessary to analyze the effects of shearing on vortex structure identification and vortex dynamics.

One of the latest approaches, which is considering the effects of shear in the vortex identification, is the approach of Kolář [3]. The triple decomposition method of Kolář is an extension of the classical decomposition of the velocity gradient tensor  $\nabla \mathbf{u}$  into a symmetric and an asymmetric part. The motivation of the new decomposition is the fact that the asymmetric tensor, which holds the vorticity, is not able to distinguish between pure shearing motion and vortical motion. Therefore, the de-

composition of  $\nabla \mathbf{u}$  into three tensors is desirable, where the third tensor describes pure shear motion and the new *corrected* vorticity tensor considers only vortical motion. Although the application of the new criterion for two-dimensional examples, e.g. mixing layers, is promising, the lack of an algorithm for three-dimensional flow fields makes a general application impossible, at present.

For the examples in the following sections the  $\lambda_2$  criterion is used, which is, despite its inaccuracies in high-shear regions, the most reliable criterion for threedimensional flow fields.

#### **3** Visualization of Vortices

The goal of our project was to provide an interactive visual representation of vortex structures. Therefore, the following subgoals should be accomplished: (1) vortex segmentation. This step is crucial for separating arbitrarily connected vortex structures, like those resulting from conventional isosurface- generation algorithms; (2) vortex visualization. Based on the foregoing segmentation step, the vortices need to be visualized appropriately; (3) multi-field visualization, i.e., the visualization of vortices and additional quantities, e.g., the velocity field; (4) vortex tracking. Since most of the underlying velocity fields represent a time-dependent flow, an appropriate time-dependent visualization of vortex structures is needed.

## 3.1 Vortex Segmentation

The role of vortex segmentation can be illustrated by a simple scenario. Imagine we have a scalar field like the one that results from the  $\lambda_2$  computation. Then, from the point of view of visualization there are various methods for drawing an isosurface of this input data, which represents, according to the theory of  $\lambda_2$ , the vortex boundaries if the isovalue is set to a small value below zero in order to avoid artifacts due to noisy data.

Our goal was to provide the engineer more interactivity in terms of selecting single vortex structures, fading them out, or simply to investigate them in more detail. Therefore, a criterion, which delivers a list of segmented vortices must be found-the vortex core line. We define a vortex core line as a connected set of *local* pressure (or  $\lambda_2$ ) minima. The end points of a vortex core line are found by searching for discontinuities of the core line tangent or by checking if the function value of the core line becomes positive. A predictor-corrector method by Stegmaier et al. [8] is used for computing the vortex core lines: first a list of seed points is generated. Since 3D minima of  $\lambda_2$  are a subclass of the local  $\lambda_2$  minima discussed above, they must be part of the vortex core line and, therefore, can serve as seed points without any restrictions.

Once the list of seed points has been generated, the algorithm starts by processing them in a consecutive order. Each seed point is tested if it is inside the vortex boundaries, the  $\lambda_2 = 0$  isosurface, of an already detected vortex. If this is the case, the seed point is discarded. Otherwise it serves as first point  $\mathbf{p}_0$  on the growing vor-



Fig. 1 Visualization of a segmented vortex field: Figure (a) shows the  $\lambda_2$  isosurfaces colored according to the vortex strength. Here, the gradient goes from green (= weak) to red (= strong); Figure (b) demonstrates the interactivity of our framework. Individual vortices can be selected and transformed while uninteresting vortices are blended out and drawn as small quadrilaterals at the top of the screen.

tex core line, where both a prediction and a correction step are performed. Since the vorticity can be regarded as the local axis of rotation and we assume that the vortex rotates around the vortex core line, it can be used for predicting the next position on the core line. However, an integration along the vorticity vector would involve a numerical error, which need to be corrected in a second step. Here, again the vorticity is computed, now at the predicted position. The corrected position  $\mathbf{p}_{i+1}$ is finally found by searching for the  $\lambda_2$  minimum on a plane perpendicular to the vorticity. Then the loop is repeated, using  $\mathbf{p}_{i+1}$  as starting point until a discontinuity of the core line is detected or  $\lambda_2$  becomes positive. In order to identify all vortex core lines, the algorithm is performed over all seed points. For a more detailed discussion we refer to [8].

In a last step, the isosurfaces are computed for each vortex individually by creating planes at the computed positions  $\mathbf{p}_i$ . The geometrical data is stored together with the points and the topology of the vortex core lines into appropriate data structures, which are discussed in Section 3.2.

# 3.2 Vortex Visualization

Once the vortices and their respective core lines are computed, they to need be stored into data structures which are appropriate for an interactive visualization. Here, the goal of vortex visualization exceeds a simple visualization of their  $\lambda_2$  isosurfaces—apart from a good visual representation, the user should be able to interact with the visualized data. Therefore, the following features should be supported: (1) global transformation of all vortices; (2) selection and transformation of individual vortex structures; and (3) removal of uninteresting vortices.



Fig. 2 Line integral convolution (LIC)—a texture is smeared out in direction of the vector field. This results in low frequencies along the stream line and high frequencies perpendicular to its gradient: (a) shows LIC on a 2D planar surface with additional dye advection; (b) LIC has been applied on a curved surface, here the  $\lambda_2$  isosurface representing a vortex region. The vector field is projected onto the isosurface to achieve long line-like patterns; (c) the shape of the particle traces are further emphasized by enabling illumination.

Although the first condition could also be fulfilled when only  $\lambda_2$  isosurfaces would be drawn, for (2) and (3) the vortex segmentation of Section 3.1 is absolutely crucial. Since (2) and (3) intend to perform any actions on individual vortex structures, the most appropriate data representation would consist of a scene graph, i.e., a data structure, which stores each element of the scene individually and where for each scene element a separate transformation matrix is attached. In this case, the vortex core lines and their related  $\lambda_2$  isosurfaces build the leaves of the scene graph and drawing can be performed for each leaf individually.

Drawing the vortices in a consecutive order with one global transformation matrix fulfills condition (1); enabling the local transformation matrices additionally allows the transformation of single vortex structures and, therefore, fulfills condition (2); condition (3) exploits again the fact that the vortices are drawn in a consecutive order. If a vortex should not be drawn, a small quad is created at the upper part of the screen and represents the invisible vortex. Figure 1 shows the vortex visualization, where the colors represent the vortex strength according to the  $\lambda_2$  values on the vortex core line. In Figure 1 (a), the segmented vortices of a transitional flow are shown. Figure 1 (b) illustrates how an individual vortex is picked, while uninteresting vortices are faded out and displayed as small quadrilaterals in the upper left corner of the window. Due to the underlying scene graph, the selected vortex can be transformed independently. Applying color coding to the isosurfaces (green = weak, red = strong) helps the user to identify stronger vortices at a glance.

#### 3.3 Introducing Line Integral Convolution

So far, vortices have been extracted from a velocity field, vortex core lines were computed, and an appropriate visualization framework was developed in order to enable an interactive investigation of the extracted vortices. However, up to now, only the locations, the spatial extents, and the vortex strengths are perceptible for the user—there is no possibility to identify the direction of rotation of the individual vortices so far! This information, though, is quite important because it plays a crucial role in vortex dynamics, where the direction of rotation of a vortex is of special meaning considering the amplification of other vortices (see Section 5).

For this purpose, the flow feature visualization described above is combined with a real-time texture-based flow visualization technique-the line integral convolution (LIC) algorithm [1]–which is a common method in computer graphics. The major advantage of this algorithm is that it exploits the capabilities of modern graphics hardware and, therefore, is highly interactive. Without going into technical detail, LIC is actually used to draw stream lines by integrating along a given vector field, whereby stream lines appear as lines of low frequencies. A result of 2D LIC is shown in Figure 2 (a). Since the original LIC algorithm is not appropriate for drawing stream lines on curved surfaces as those which result from our vortex visualization algorithm of section 3.2, an extended version by Weiskopf and Ertl [9] has been applied to the  $\lambda_2$  isosurfaces. Based on the fact that the velocity field is almost tangential to the  $\lambda_2 = 0$  isosurface, the vector field is projected onto the isosurface. This results in line-like patterns on the  $\lambda_2$  isosurfaces and has the following advantages: (1) the motion of a particle close to a vortex region is perceptible at a glance; and (2) applying animated LIC, the direction of rotation is perceptible as well without loosing the information provided by the vortex visualization. Figure 2 (b) and (c) show LIC on vortex surfaces, whereby in (c) the particle traces are further emphasized by illumination. For a more detailed discussion we refer to the work of Schafhitzel et al. [5].

#### 4 Vortex Tracking

Considering unsteady vector fields instead of individual snapshots, the temporal behavior of the extracted vortex structures are of special interest because this information can be used for detecting temporal events. The mapping of vortex structures, which have been extracted at time *t* to their counterparts at a later time step  $t + \Delta t$  is called vortex tracking and builds the base of any temporal investigation of vortices.

The tracking algorithm we used is based on the work of Schafhitzel et al. [6], where the vortex core line at time *t* is sampled and particles are integrated along the vector field in order to estimate the new position of the vortex core line at time  $t + \Delta t$ . This method is simple and reliable, since the vortex core line is supposed to be the region inside a vortex which exceeds at least the upper boundary of  $\lambda_2$  ( $\lambda_2 = 0$ ) and, therefore, the particles seeded on the vortex core lines do not tend to leave the vortex from *t* to  $t + \Delta t$ .

Our framework, which is represented in Figure 3, supports user interactivity in the following manner: (1) the user is able to select a single vortex at an arbitrary time step in order to track it forward and backward along the time-dependent vector field. Here, all the relationships between the vortices at the different time steps are created; (2) a time line is provided, which can be used to manipulate the time at which the vortex should be visualized. By smoothly sliding the red bar along the time



**Fig. 3** The visualization of an experimental data set [4]. The data was measured using tomographic PIV and shows a circular cylinder wake at Reynolds Re = 360. The cylinder, which is located parallel to the left edge of the data set is not visualized. In this example, vortex dynamics are considered: (a) initial situation at t = 0. All vortices are color coded according to their strength. The red bounding box gives feedback about the vortex selected for tracking; (b) the moment of strongest perturbation at  $t_p = 4$ . The representation has changed to time  $t_p$  where the green vortex influences the selected structure (red); (c) backward tracking. The wire frame representation shows where the structures came from; (d) forward tracking. Here, the wire frame representation shows the further development of both structures [6]. ©2008 by Schafhitzel et al.

line, the transitions between the time steps are visualized; (3) topological changes are supported when a vortex core line hits more than one structure. Note that the detection of vortex reconnection events depends on the quality of the underlying vortex core line detection method. For a topology-preserving  $\lambda_2$ -based vortex core line detection method we refer to the work of Schafhitzel et al. [7]. Altogether, this method provides a fast and interactive investigation of vortices inside a time-dependent flow because once a vortex has been selected and tracked, the user is able to switch between the different time steps very fast.

# **5** Considering Vortex Dynamics

The analysis of coherent structures is based on the purpose to gain physical understanding of the mechanisms in fluid dynamics, which is necessary for a reliable comparability of experimental and numerical studies, the reproducibility of these studies, or predictive modeling of flow field regions, e.g. separated flows and wakes. A method for the detection of vortex core lines is described in Section 3.1; the tracking of vortices is discussed in Section 4. These methods enable the spatial and temporal investigation of kinematic properties (e.g. size, vorticity, and energy) of vortex structures and, therefore, provide quantitative information for an advanced comparison of studies and physical models.

However, an analysis of coherent structures, which is limited to kinematic properties cannot be sufficient in the investigation of central questions in the development of flow fields. Hence, an additional investigation of the dynamic properties (e.g. emergence, growth, and stability) of the coherent structures is necessary to answer these questions.

A straightforward strategy for the consideration of vortex dynamics in the analysis of coherent structures is the observation of dynamical values, e.g., kinetic energy or enstrophy, which are defined by

$$E_{kin} = \frac{1}{2} \int_{V} \left( \mathbf{u} \left( \mathbf{x}, t \right) \right)^{2} d^{3} \mathbf{x} \text{ and } E_{rot} = \frac{1}{2} \int_{V} \boldsymbol{\omega} \left( \mathbf{x}, t \right)^{2} d^{3} \mathbf{x} , \qquad (1)$$

respectively, where  $\omega$  denotes vorticity. Thereby, several problems arise: the first is the lack of a proper definition of the region of integration. In test cases with two vortex structures like collision of vortex rings, it is sufficient to integrate over the whole domain. However, in real flow fields containing hundreds of structures of interest, each of them needs a more specified definition of the integrated volume. A reasonable choice are the spatial extents of the vortical regions.

The second problem addresses a major aspect in the investigation of dynamical properties-the interaction between coherent structures-like the merging of coherent structures or the self-excited effects of complex coherent structures (e.g. hairpin vortices). A technique which deals with these effects is the determination of the induced velocities of each coherent structure. The fundamental equation for this technique is the Biot-Savart equation, which computes the induced velocity field out of the vorticity field:

$$\mathbf{u}_{ind}(\mathbf{x},t) = \frac{1}{4\pi} \int_{V} \frac{\boldsymbol{\omega}(\mathbf{x}',t) \times (\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} d^3 \mathbf{x}' , \qquad (2)$$

where the influence of  $\mathbf{x}'$  on  $\mathbf{x}$  is computed.

A pure monitoring of the induced velocities has only a limited significance for a characterization of the interaction between coherent structures because the consideration of the influences on a vector field is not straightforward. Hence, an approach, which combines both methods described above is used to determine the interaction of coherent structures. For this purpose, the influence of one vortex structure on an-



**Fig. 4** Program flow of our framework: first, the vortex core lines are computed; second, the structures are tracked and computed for their influence on each other. This stage can be interactively manipulated by the selection of the structures to be tracked [6]. ©2008 by Schafhitzel et al.

other is described by a characteristic integral value, e.g. the kinetic energy, which depends on the induced velocity field  $\mathbf{u}_{ind}$ . Figure 4 shows the overall program flow which is separated into two independent parts: (1) the preprocessing, where the core lines and the vortex segmentation are computed; and (2) the real-time processing, which consists of the tracking and the consideration of vortex dynamics. In contrast to the first stage, the second stage can be widely influenced by the user.

The example presented in Figure 3 shows results of the tracking method, which has been extended by taking into account vortex dynamics in order to find the time where the selected structure is perturbed strongest by another vortex. The computation results in a list containing the strongest perturbing vortices and the appropriate information in terms of kinetic energy and enstrophy.

The resulting time instant  $t_p$  defines the time of the strongest perturbation, and therefore, the visualization is set to  $t_p$  for both the focus and the context. In Figure 3, the detection and visualization of  $t_p$  is shown. By selecting a vortex at an arbitrary time (Figure 3(a)), the vortex with the strongest influence is computed, displayed as green vortex in Figure 3(b) at time  $t_p$ . Starting from here, both structures are tracked backward and forward to gain information about their origin and their further development (Figures 3(c) and (d)). Additionally, the detailed information about the detected and the selected vortex, e.g., the energy induced, IDs, length, etc. is displayed as text.

A detailed investigation of areas of interest inside a flow field or selected vortex structures as described in Figure 3 reveals additional temporal information about interactions between vortex structures and their effects on flow-field events. This helps the researcher to perform detailed studies of unsteady flow fields like, e.g. the wake of a circular cylinder, that has been selected as a reference configuration for this DFG priority research area. However, in absence of suitable data from other projects we had to work on data provided by Scarano et .al [4] for the present example.

## 6 Conclusions

Recent flow-field measurements techniques such as those considered within the present DFG priority research program are capable of producing several gigabytes of raw data within a short time. Without appropriate post-processing and visualization methods the physical insight hidden in these data would remain unexplored. The methods presented here allow the researcher an interactive exploration of such data on a higher abstraction level based on the concept of "vortices". As such, vortex identification, segmentation, quantification and tracking are a prerequisite for any detailed investigation of *unsteady* separated flows, which might be encountered after boundary layer separation, in mixing layers or in wakes. However, the present tool is not yet complete,. It still lacks more methods for an automatic tracking of events like vortex creation out of shear or vortex merging. According work is necessary and in progress.

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