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A method for the identification of high-frequency oscillations in unsteady flows

In direct numerical simulations (DNS) of instabilities related to laminar-turbulent transition the identification of oscillations with small amplitudes is very difficult. Especially in laminar separation bubbles slight or in some cases even strong time variations of the base flow dominate the Fourier transform for all frequencies. We show that higher-order derivatives of the time signal amplify high-frequency components and low-frequency trends are almost removed. Thus, time derivatives are well suited to improve results of Fourier transforms using a standard Hanning window.

1. Laminar separation bubbles

Under certain conditions laminar separation bubbles are unsteady. The bubble increases and decreases in size periodically. Typical time signals of the wall vorticity (a measure for the shear stress, figure 1a) for such a case show two time scales:

- a slow oscillation of the base flow with the frequency f_{bf} (period T_{bf}).
- high-frequency oscillations, so-called Tollmien-Schlichting (TS) waves, $f_{TS} \approx 100 f_{bf}$, which have only temporarily large amplitudes (time window B) and otherwise no influence on the base-flow (A).

2. Using time derivatives to improve the Fourier transform

A Fourier transform is generally well suited for the detection of TS-waves due to its harmonic ansatz. For the given time signal, however, it is impossible to fulfill the underlying assumptions in a satisfying manner:

- the time window has to be a complete multiple of the period of any disturbance, and resulting from that:
- the discontinuity of the time signal and its derivatives at the borders must be low. Otherwise, this discontinuity causes non-realistic large-amplitude contributions to the analysis for all frequencies.

Applying a window function, as for example a Hanning window: $v_{han}(t) = v(t)[0.5 - 0.5 * cos(2\pi(t - t_a)/(t_e - t_a))]$ helps to reduce the problems caused by the borders.
To further improve the method, we analyse derivatives of the time signal taking into account the different characteristics of the unsteadiness in the data (see above). For harmonic waves with the amplitude A: $v = A * e^{i\beta t}$, the nth time derivative is $v^{(n)} = (i\beta)^n v$. TS-waves are harmonic in time, whereas the low-frequency behaviour is supposed to be a mixture of both polynomial and harmonic components. Since the nth and higher derivatives of polynomials of the order n are zero, low-frequency trends in the time signal are expected to be considerably reduced. Harmonic components are amplified by the factor $i\beta^n$. Thus, oscillations with high frequencies are strongly magnified compared with low-frequency components. However, this effect can be corrected by recalculating the Fourier amplitude of the n$^t h$ derivative $A_\beta^{(n)}$ by: $A_\beta = A_\beta^{(n)}/\beta^n$.

3. Examples

The effect of using time-derivatives for better amplitude resolution in a Fourier transform will be demonstrated by two simple examples (figure 2). Two harmonic oscillations are superposed – a high-amplitude low-frequency disturbance D_1 and a low-amplitude high-frequency disturbance D_2. Time signals and derivatives are calculated analytically.
Figure 2a) examines the influence of a time window that does not exactly match the period T_1 of the signal. Due to the discontinuity at the borders of the time window, an error is spread into the whole frequency regime which is decreasing with increasing frequency. The error thus introduced is bigger for a sine wave than for a cosine wave due to the steeper descent and hence stronger discontinuity at the boundaries. Although the time window fits almost the period T_1, the disturbance D_2 cannot be detected even if a Hanning window is employed . Using fourth derivatives (dashed lines) improves the amplitude resolution by approximately eight decades in the high-frequency regime and D_2 appears as a peak in the spectrum.

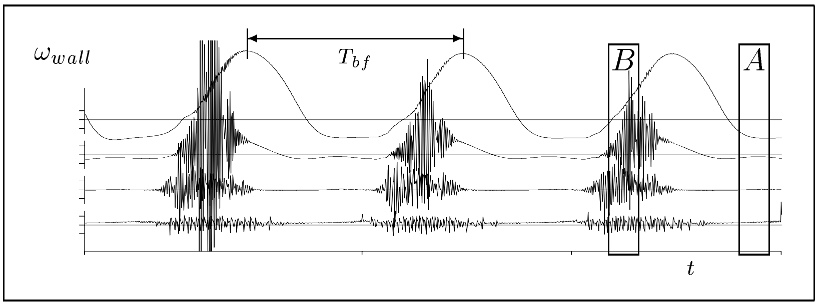

Figure 1: DNS wall vorticity: Time signals at different downstream positions x (increasing from top to bottom).

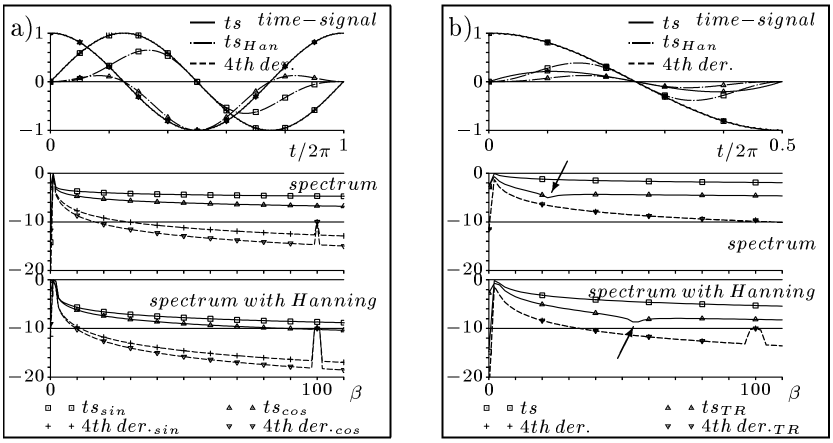

Figure 2: Superposition of disturbances D_1 and D_2: $A_1 = 1$, $A_2 = 10^{-10}$, $\beta_2 = 100$. Top: time signal without/ with Hanning window (ts/ts_{Han}) and fourth derivatives (4th der.);
Center/Bottom: spectrum without/ with Hanning window.
a) low-frequency ($\beta_1 = 1.001$) sine and cosine wave with a time window slightly too large ($\beta_1 = 1.001$; instead of 1).
b) large discontinuity at the borders: time window = $1/2\,T_1$ ($\beta_1 = 1$).
TR = trend removal; results in incorrect dents (arrows) in the spectrum.

To examine the effect of time derivatives on a time window with a strong trend of the mean flow, the duration of the time window is chosen to be one half of the low-frequency period T_1 in figure 2b). As expected, the results of a standard Fourier transform are even worse in this case. Employing a trend-removal technique (forcing the signal to zero at the borders) and applying a Hanning window is not sufficient to detect the low-amplitude disturbance, whereas applying fourth derivatives is again much better. Additionally, the trend-removal technique introduces 'unphysical' dents into the spectrum at different frequencies, depending on whether a Hanning window is applied or not. Such dents might be misinterpreted as being inherent in the time signal.
To demonstrate the **practical relevance** of the proposed method, the amplification curves of the simulation data in time window A (see figure 1) are compared to the original technique in figure 3. Downstream of $x \approx 3.6$, the Fourier transform agrees very well with the new technique based on the fourth derivatives (∂t^4). However, upstream of $x \approx 3.6$, no meaningful result can be attributed to the time signal due to the problems discussed above. The curves of the fourth derivatives instead show a very good quantitative agreement if compared with linear stability theory.

4. Conclusions

Many technical and physical problems are dominated by combinations of harmonic oscillations. Their analysis is difficult if frequencies and amplitudes differ strongly. In this case, applying higher-order derivatives can improve the amplitude resolution of the Fourier-transform. The method is easy to implement, and besides employing a window-function, it requires no additional signal processing and the result is independent of empirical parameters (e.g. filters). Nevertheless, this method is only applicable for harmonic high-frequency oscillations and requires very accurate time-signals (otherwise numerical errors or measurement errors are strongly enhanced also).

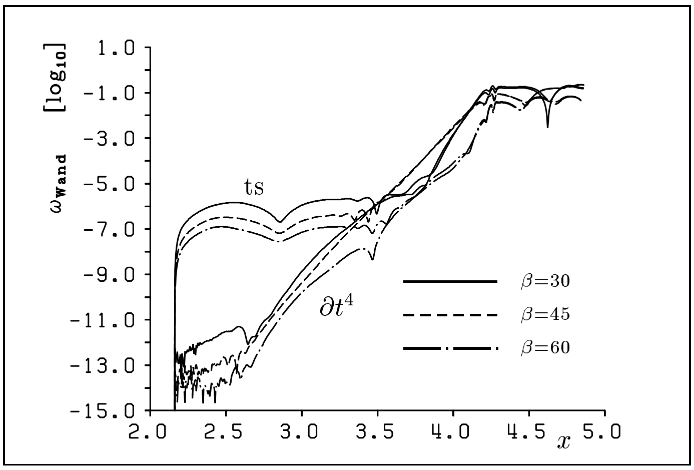

Figure 3: Amplification curves from window A in figure 1.

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